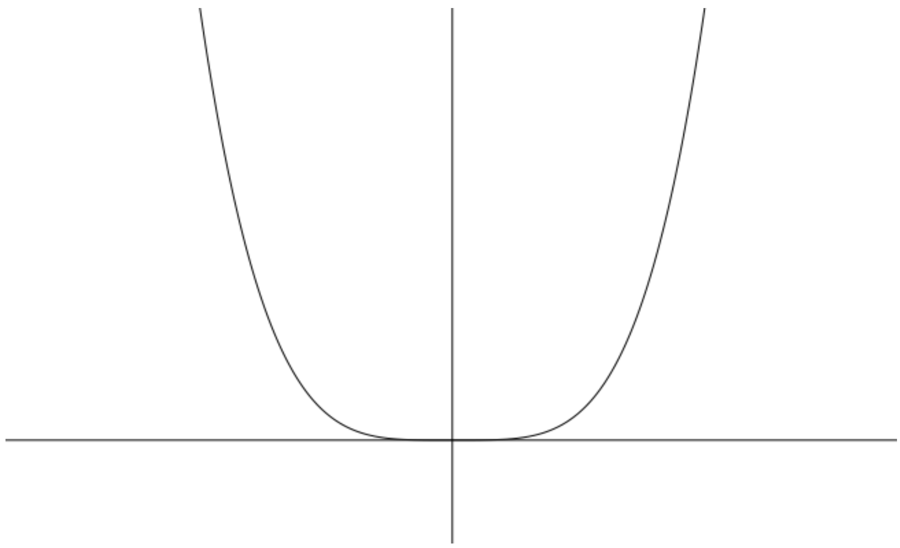
Section 4.3 --- Other Common Functions

Power functions of the form $y = ax^n$ with $n = 2, 4, 6, \ldots$

The graphs of $y = x^2, y = x^4, y = x^6, \ldots$ are U-shaped curves opening upward with the turning point at the origin.



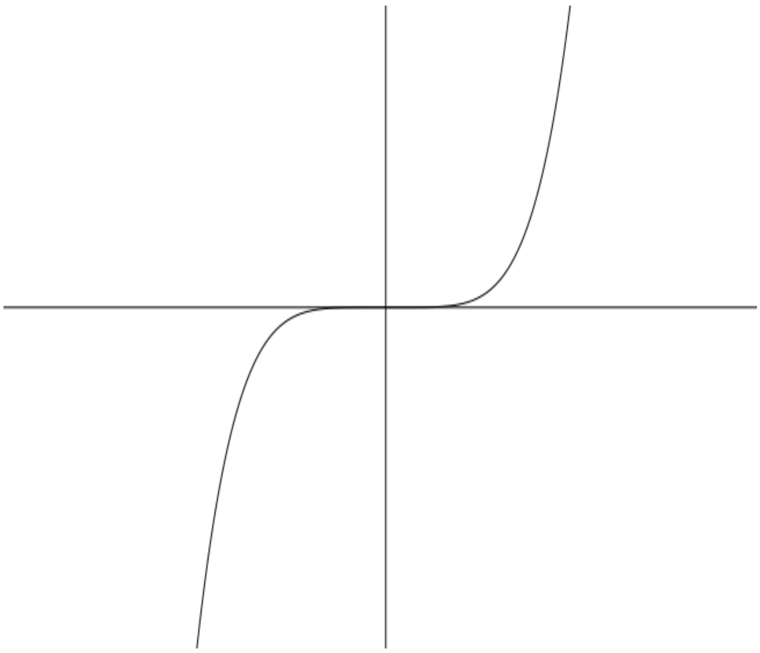
The bigger the number n, the flatter the curve is on the interval $(-1,1)$ and the steeper the curve is outside that interval.

Multiplication by a to form $y = ax^n$ has three possible effects:

- If $|a| > 1$, the graph is stretched vertically.
- If $0 < |a| < 1$, the graph is compressed vertically.
- If $a < 0$, the graph is reflected about the x-axis.

Power functions of the form $y = ax^n$ with $n = 3, 5, 7, \ldots$

The graphs of $y = x^3, y = x^5, y = x^7, \ldots$ have the following shape. Each has a flat spot at the origin.

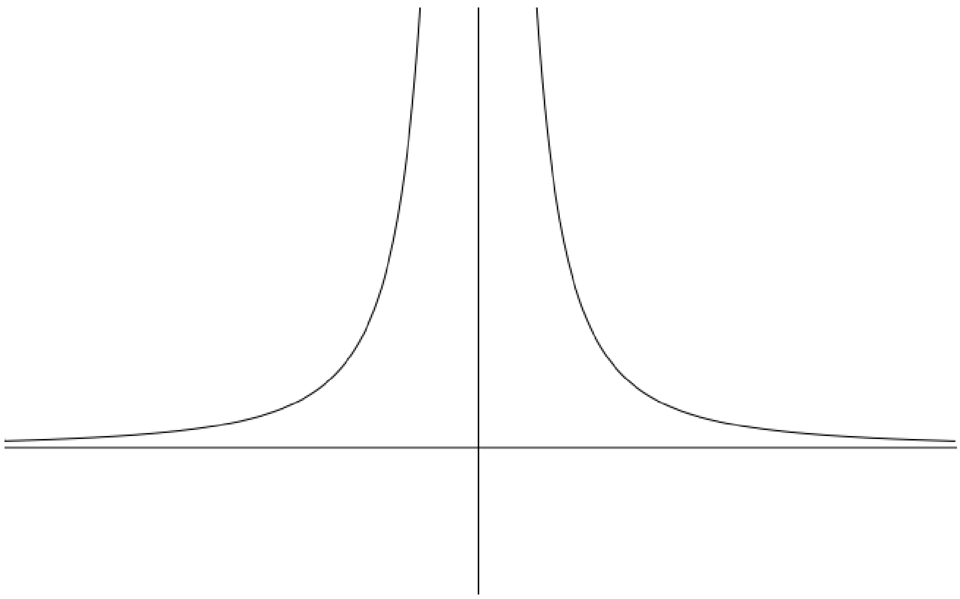


The bigger the number n, the flatter the curve is on the interval $(-1,1)$ and the steeper the curve is outside that interval.

The effects of multiplication by a number a are the same as above.

Reciprocal functions of the form $y = a/x^n$ with $n = 2, 4, 6, \ldots$

The graphs of $y = 1/x^2, y = 1/x^4, y = 1/x^6, \ldots$ have the following shape.



The bigger the number n, the more square the round corner becomes.

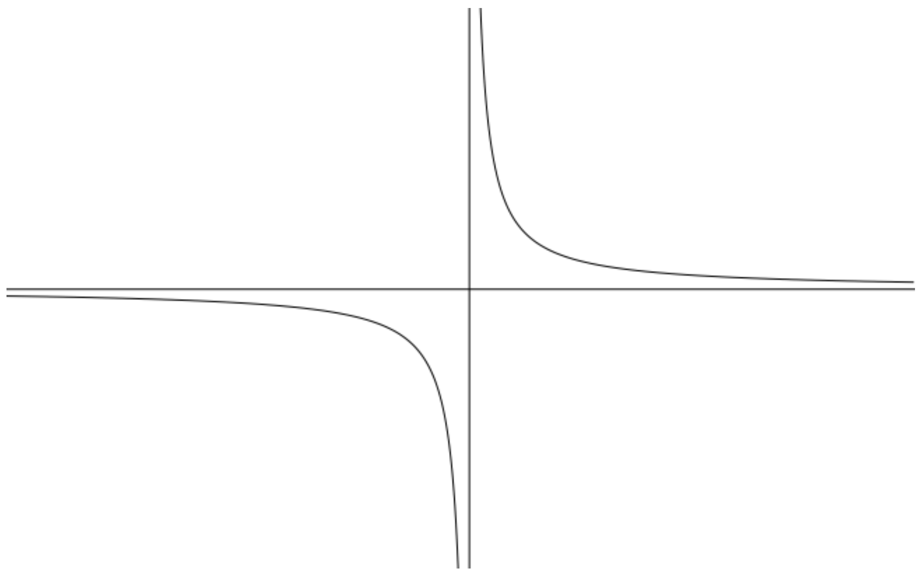
These functions have the following properties:

- Not defined at $x = 0$. In fact, the vertical line $x = 0$ is a vertical asymptote of the graph.
- $y > 0$ for all valid x-values. In fact, the horizontal line $y = 0$ is a horizontal asymptote of the graph.

The effects of multiplication by a number a are the same as above.

Reciprocal functions of the form $y = a/x^n$ with $n = 1, 3, 5, \dots$

The graphs of $y = 1/x, y = 1/x^3, y = 1/x^5, \dots$ have the following shape.



The bigger the number n, the more square the round corner becomes.

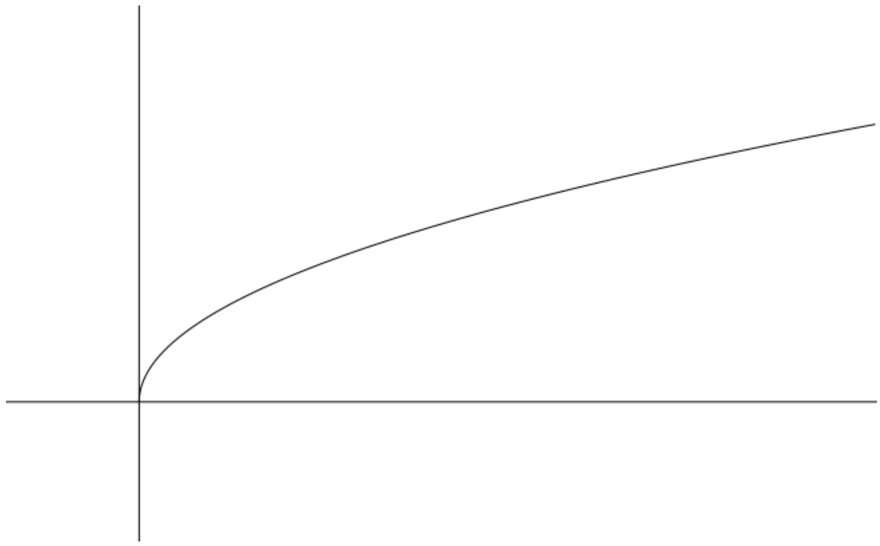
These functions have the following properties:

- Not defined at $x = 0$. In fact, the vertical line $x = 0$ is a vertical asymptote of the graph.
- The horizontal line $y = 0$ is a horizontal asymptote of the graph.

The effects of multiplication by a number a are the same as above.

Radical functions of the form $y = a\sqrt[n]{x}$ with $n = 2, 4, 6, \ldots$

The graphs of $y = \sqrt{x}, y = \sqrt[4]{x}, y = \sqrt[6]{x}, \ldots$ have the following shape.



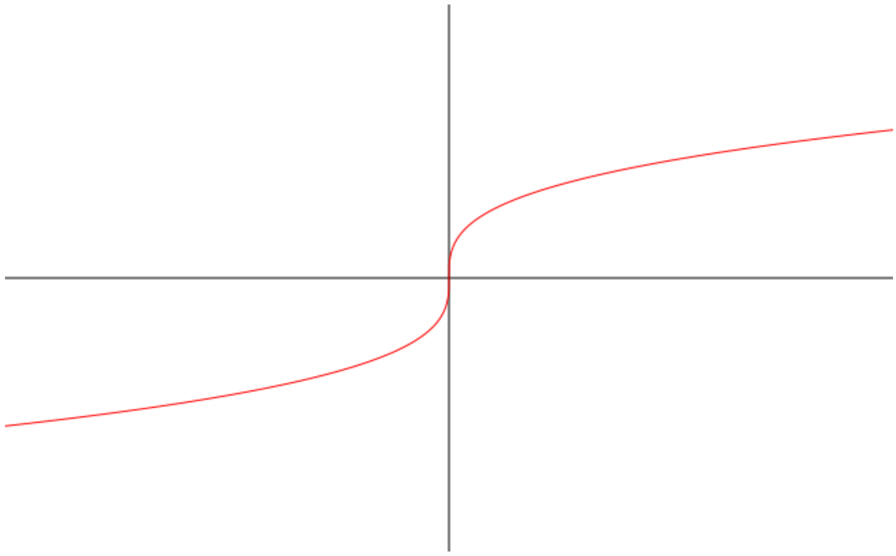
These are the graphs of the even-power functions turned sideways (and cut in half). The bigger the number n, the more square the round corner becomes.

The effects of multiplication by a number a are the same as above.

VERY IMPORTANT IDEA: Recall that $\sqrt[n]{x^m} = x^{m/n}$.

Radical functions of the form $y = a\sqrt[n]{x}$ with $n = 3, 5, 7, \ldots$

The graphs of $y = \sqrt[3]{x}, y = \sqrt[5]{x}, y = \sqrt[7]{x}, \ldots$ have the following shape.



These are the graphs of the odd-power functions turned sideways and flipped.
The bigger the number n, the more square the round corners become.

The effects of multiplication by a number a are the same as above.

Piecewise-defined functions...

A **piecewise-defined function** is a function defined by two or more formulas, where valid particular formula is valid only for a unique portion of the function's overall domain.

Example

$$f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ 3x - 6, & x > 2 \end{cases}$$

When evaluating or graphing piecewise-defined functions, be sure to use only the formula that applies to the portion of the domain that is under consideration.

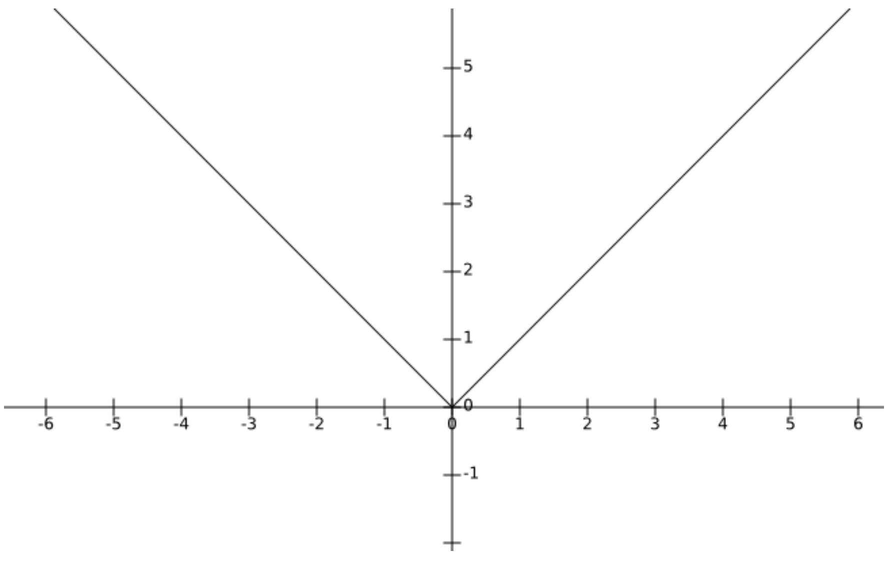
A function is said to be **continuous** if its graph can be sketched without picking up your pencil. Piecewise-defined functions can be discontinuous at the break points even if the individual formulas define continuous functions.

Absolute value function...

The **absolute value function** is a very common piecewise-defined function. In fact, it is so common that it is given its own special notation:

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Notice that the absolute value function never gives negative values. Its graph is a combination of the graphs of $y = x$ and $y = -x$ over their respective portions of the domain.



The effects of multiplication of $|x|$ by a number a are the same as those described above.