1. Differentiate. Do not simplify. $\frac{d}{dx}\left[\frac{\tan x}{x^4 + 5x^2}\right]$

(a) $\dfrac{\sec^2 x}{4x^3 + 10x}$

(b) $\dfrac{(\tan x)(4x^3 + 10x) - (x^4 + 5x^2)(\sec^2 x)}{(x^4 + 5x^2)^2}$

(c) $\dfrac{(x^4 + 5x^2)(\sec^2 x) - (\tan x)(4x^3 + 10x)}{(x^4 + 5x^2)^2}$

(d) $\dfrac{(x^4 + 5x^2)(\sec^2 x) - (\tan x)(4x^3 + 10x)}{\tan^2 x}$

Solution

2. Differentiate. Do not simplify. $\frac{d}{dx}[(x^2+2x+3)\cos x]$

(a) $(2x+2)\cos x-(x^2+2x+3)\sin x$

(b) $-(2x+2)\sin x$

(c) $(x^2+2x+3)\cos x-(2x+2)\sin x$

(d) $(x^2+2x+3)(-\sin x)+(x^2+2x+3)(\cos x)$

Solution

3. Which one of these is NOT the chain rule?

(a) $\dfrac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$

(b) $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

(c) $\dfrac{d}{dx}(f \circ g)(x) = (f \circ g')(x) \cdot g'(x)$

(d) $\dfrac{d}{dx}[f(x)g(x)] = f'(g'(x)) + g'(f(x))$

Solution

4. Find the slope of the line tangent to the graph of $y = \sqrt{x^3 - x - 2}$ at the point where $x = 2$.

(a) 11

(b) 11/4

(c) $\sqrt{11}/22$

(d) The slope is not defined.

Solution

5. Differentiate. $\frac{d}{d\theta} \sin^2(3\pi\theta)$

(a) $6\pi \sin(3\pi\theta) \cos(3\pi\theta)$

(b) $6\pi \cos(3\pi\theta)$

(c) $2 \cos(3\pi\theta)$

(d) $2 \sin(3\pi\theta) \cos(3\pi\theta)$

Solution

Problem 1 — The answer is (c).

Using the quotient rule, the derivative is
Low D High minus High D Low all over Low squared.

$$\frac{d}{dx}\left[\frac{\tan x}{x^4 + 5x^2}\right] = \frac{(x^4 + 5x^2)(\sec^2 x) - (\tan x)(4x^3 + 10x)}{(x^4 + 5x^2)^2}$$

Back to Problem 1

Problem 2 — The answer is (a).

Using the product rule,

$$\frac{d}{dx}[(x^2+2x+3)\cos x] = (2x+2)\cos x + (x^2+2x+3)(-\sin x)$$

Back to Problem 2

Problem 3 — The answer is (d).

Choices (a), (b), and (c) all are forms of the chain rule. Choice (d) looks a bit like the product rule, but the formula is not correct.

Back to Problem 3

Problem 4 — The answer is (b).

The slope of the tangent line is given by the derivative.

$$\frac{dy}{dx} = \frac{1}{2}(x^3 - x - 2)^{-1/2}(3x^2 - 1) = \frac{3x^2 - 1}{2\sqrt{x^3 - x - 2}}$$

$$\frac{dy}{dx} \text{ at } 2 = \frac{11}{4}$$

Back to Problem 4

Problem 5 — The answer is (a).

$$\frac{d}{d\theta}\sin^2(3\pi\theta) = 2\sin(3\pi\theta)\cdot\frac{d}{d\theta}\sin(3\pi\theta)$$

$$= 2\sin(3\pi\theta)\cos(3\pi\theta)(3\pi)$$

Back to Problem 5