1. Evaluate the following limit.

$$\lim_{x \to \infty} \frac{7x^3 - 5x^2 + 8}{6x^3 + 9x^2 - 13x}$$

(a) $7/6$

(b) $-8/13$

(c) ∞/∞

(d) 0

Solution

2. Evaluate the following limit.

$$\lim_{x \to -\infty} \frac{8x^4 - 7x^2 + 6x}{x + 12}$$

(a) 8

(b) 1/2

(c) $-\infty$

(d) 0

Solution

3. Determine the horizontal asymptote of the graph of R.

$$R(x) = \frac{x^2 + x - 8}{5x^3 + 7x^2 + 2}$$

(a) $y = 0$

(b) $y = 1/5$

(c) $y = -4$

(d) The graph has no horizontal asymptote.

Solution

4. Evaluate the following limit.

$$\lim_{x\to\infty} \frac{x+1}{(x^2+1)^{1/3}}$$

(a) 1

(b) ∞

(c) ∞/∞

(d) 0

Solution

5. Evaluate the following limit.

$$\lim_{x \to \infty} \frac{x - \cos x}{x}$$

(a) 0

(b) ∞

(c) 1

(d) The limit does not exist.

Solution

Problem 1 — The answer is (a).

The highest power of x in the denominator is x^3.

$$\lim_{x\to\infty} \frac{7x^3 - 5x^2 + 8}{6x^3 + 9x^2 - 13x} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{7 - (5/x) + (8/x^3)}{6 + (9/x) - (13/x^2)} = \frac{7}{6}$$

Back to Problem 1

Problem 2 — The answer is (c).

The highest power of x in the denominator is x.

$$\lim_{x\to-\infty} \frac{8x^4-7x^2+6x}{x+12}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty} \frac{8x^3-7x+6}{1+(12/x)}$$

$$= \lim_{x\to-\infty} x(8x^2-7)+6 = -\infty(\infty)+6 = -\infty$$

Back to Problem 2

Problem 3 — The answer is (a).

R is a rational function. The degree of the numerator is 2, while the degree of the denominator is 3. Since the denominator has higher degree than the numerator, $y = 0$ is the horizontal asymptote.

Back to Problem 3

Problem 4 — The answer is (b).

The highest power of x in the denominator is $x^{2/3}$.

$$\lim_{x\to\infty} \frac{x+1}{(x^2+1)^{1/3}} \cdot \frac{\frac{1}{x^{2/3}}}{\frac{1}{x^{2/3}}} = \lim_{x\to\infty} \frac{x^{1/3}+(1/x^{2/3})}{(1+(1/x^2))^{1/3}} = \frac{\infty+0}{1} = \infty$$

Back to Problem 4

Problem 5 — The answer is (c).

$$\lim_{x\to\infty} \frac{x - \cos x}{x} = \left(\lim_{x\to\infty} \frac{x}{x}\right) - \left(\lim_{x\to\infty} \frac{\cos x}{x}\right) = 1 - 0 = 1$$

Back to Problem 5