1. Given this table of values, is it possible that $\lim_{x \to 0} f(x)$ DNE?

x	-0.1	0.1	-0.01	0.01	-0.001	0.001
$f(x)$	2.8	3.2	2.98	3.02	2.998	3.002

(a) Yes

(b) No

Solution

2. Assuming it exists, estimate $\lim_{x \to 3} f(x)$.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$	7.09	7.098	7.0999	15	7.1002	7.103	7.12

(a) 7

(b) 15

(c) 7.1

(d) 7.5

Solution

3. In which of the four ways does the following limit fail to exit?

$$\lim_{x \to \pi} \frac{x}{\sin^2 x}$$

(a) Failure #1

(b) Failure #2

(c) Failure #3

(d) Failure #4

Solution

4. In which of the four ways does the following limit fail to exit?

$$\lim_{x \to 5} \sqrt[4]{x - 5}$$

(a) Failure #1

(b) Failure #2

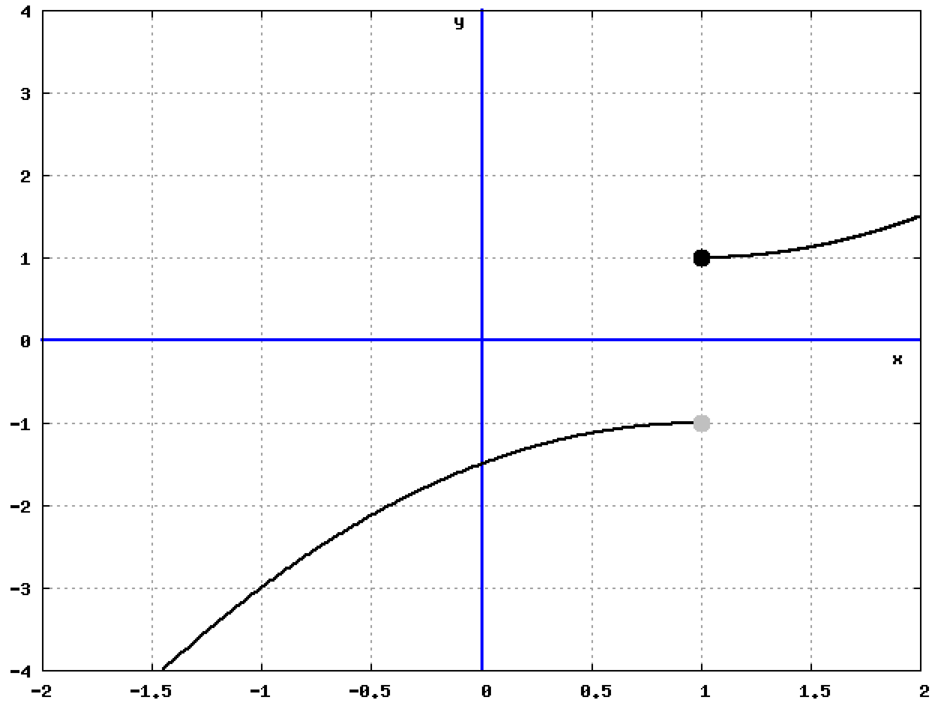
(c) Failure #3

(d) Failure #4

Solution

5. In which of the four ways does the limit at $x = 1$ fail to exit?

(a) Failure #1
(b) Failure #2
(c) Failure #3
(d) Failure #4



Solution

Problem 1 — The answer is yes (a).

The table seems to indicate that the limit might be 3. However, we can NEVER precisely determine a limit from a table. In this example, the function used to construct the table was

$$f(x) = \begin{cases} 2x + 3, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$$

In this case,

$$\lim_{x \to 0} f(x) \text{ DNE}$$

Back to Problem 1

Problem 2 — The answer is (c).

As x gets closer and closer to 3, the table seems to indicate that the function values get close to 7.1. Based on the given information, this is the best estimate. Notice that the value of f at $x = 3$, $f(3) = 15$, is completely irrelevant.

Back to Problem 2

Problem 3 — The answer is (b).

Near $x = \pi$, the numerator is nearly π, while the denominator is positive and very nearly zero. The values of $x/\sin^2 x$ grow without bound near $x = \pi$.

Back to Problem 3

Problem 4 — The answer is (d).

The function $f(x) = \sqrt[4]{x-5}$ is only defined for $x \geq 5$. Since $f(x)$ is not defined on an open interval containing $x = 5$, we cannot consider this limit. We must say that the limit DNE.

Back to Problem 4

Problem 5 — The answer is (a).

As $x \to 1$ from the left, the function values approach -1.
As $x \to 1$ from the right, the function values approach 1.
The limit does not exist.

Back to Problem 5

Four common ways limits fail to exist

1. Limit from the left is not equal to limit from the right

2. Function values grow without bound

3. Function values oscillate

4. Function is not defined on an interval around the limit point

Problem 3 Problem 4 Problem 5