1. Which one of the following would make the best u-substitution?

$$\int 6x(x^2 + 7)^4 \, dx$$

(a) $u = 6x$

(b) $u = x^2$

(c) $u = x^2 + 7$

(d) $u = (x^2 + 7)^4$

Solution

2. Which one of the following would make the best u-substitution?

$$\int \frac{8+\sqrt{x}}{\sqrt{x}}\,dx$$

(a) $u = 8 + \sqrt{x}$

(b) $u = x$

(c) $u = \sqrt{x}$

(d) $u = 1/\sqrt{x}$

Solution

3. Is a substitution necessary in order to evaluate this integral?

$$\int \csc x \cot x \, dx$$

(a) Yes

(b) No

Solution

4. Consider the definite integral shown below. After substituting $u = x^2 + x - 2$, what would be the new lower bound?

$$\int_1^3 (2x+1)(x^2+x-2)^7\,dx$$

(a) 1

(b) 10

(c) -1

(d) 0

Solution

5. What definite integral is obtained after substituting $u = \sin x$?

$$\int_0^{\pi/2} \cos x \sin^2 x \, dx$$

(a) $\int_0^{\pi/2} u^2 \, du$

(b) $\int_0^{1} u^3 \, du$

(c) $\int_0^{1} u^2 \, du$

Solution

Problem 1 — The answer is (c).

$$u = x^2 + 7 \Longrightarrow du = 2x\,dx \Longrightarrow 3\,du = 6x\,dx$$

$$\int 6x(x^2+7)^4\,dx = \int 3u^4\,du$$

Back to Problem 1

Problem 2 — The answer is (a).

$$u = 8 + \sqrt{x} \Longrightarrow du = \frac{1}{2}x^{-1/2}\,dx \Longrightarrow 2\,du = \frac{1}{\sqrt{x}}\,dx$$

$$\int \frac{8+\sqrt{x}}{\sqrt{x}}\,dx = \int 2u\,du$$

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Problem 3 — The answer is (b).

No way! Right off the top of our heads, we know a function whose derivative is $\csc x \cot x$.

$$\int \csc x \cot x \, dx = -\csc x + C$$

Back to Problem 3

Problem 4 — The answer is (d).

If $u = x^2 + x - 2$, then when $x = 1$,

$$u = (1)^2 + 1 - 2 = 0.$$

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Problem 5 — The answer is (c).

$$u = \sin x \Longrightarrow du = \cos x \, dx$$

$$x = 0 \Longrightarrow u = \sin 0 = 0 \quad \text{and} \quad x = \frac{\pi}{2} \Longrightarrow u = \sin \frac{\pi}{2} = 1$$

$$\int_0^{\pi/2} \cos x \sin^2 x \, dx = \int_0^1 u^2 \, du$$

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