

## Lecture 2: Review: Functions, Change, and Graphing

Objectives:

- (2.1) Evaluate functions.
- (2.2) Compute and interpret  $\Delta x$  and  $\Delta y$ .
- (2.3) Sketch basic graphs by hand.

### Functions

A function from a set  $A$  into a set  $B$  is a rule or correspondence that assigns to **each** element of  $A$  a **single** element of  $B$ . The set  $A$  is called the domain of the function—the domain is the set of all possible inputs. The set of all outputs is called the range of the function. The range is a subset of  $B$ , not necessarily all of  $B$ . A variable used to name a domain input is called an independent variable, while a variable used to name a range output is called a dependent variable.

Functions can be defined in many ways: in words; with diagrams, tables, or graphs; with equations; etc. Most of the functions we will study will be defined algebraically—the correspondence defining the function will be described by an equation.

Notice that the domain of a function is a defining characteristic of the function. The domain must be given! The same rule applied on different domains defines different functions. We will adopt the following convention:

**If the domain of a function is not explicitly given, we will assume the domain is the set of all real numbers that make sense in the context of the function's definition.**

**Example 1** Let  $f(x) = \frac{1}{1-x}$ .

- 1. Evaluate  $f(\frac{1}{2})$ .
- 2. Find all  $x$ -values for which  $f(x) = -5$ .
- 3. Find the domain and range of  $f$ .

*Solutions omitted.*

**Example 2** If Fred sells his Watchies for  $x$  dollars apiece, he makes a profit of  $p(x) = x^2 + 2x - 4$  dollars for each one he sells.

- 1. What is the domain of  $p$ ?

Since  $x$  represents a number of dollars, it only makes sense that  $x \geq 0$ .

- 2. Complete the square to find the range of  $p$ .

$$x^2 + 2x - 4 = (x + 1)^2 - 5 \geq -5$$

- 3. Simplify and interpret  $p(x + 1)$ .

$p(x + 1)$  is Fred's profit per Watchie after selling them for  $x + 1$  dollars apiece. This may be of interest to Fred if he is considering raising his price by \$1.

$$p(x + 1) = (x + 1)^2 + 2(x + 1) - 4 = x^2 + 2x + 1 + 2x + 2 - 4 = x^2 + 4x - 1$$

Notice that  $p(x + 1) - p(x) = 2x + 3$  is Fred's change in profit per Watchie if he raises his price \$1.

## Change

If the value of  $x$  changes from  $x = x_{old}$  to  $x = x_{new}$ , the change in  $x$  is denoted by  $\Delta x$ :

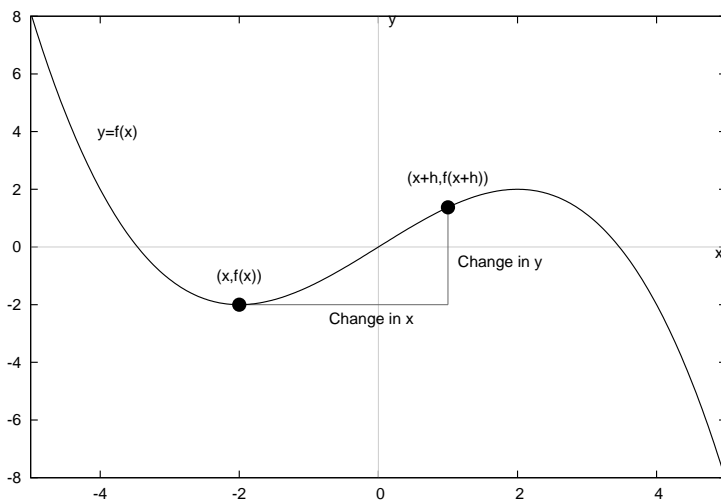
$$\Delta x = x_{new} - x_{old} \quad \text{and} \quad x_{new} = x_{old} + \Delta x.$$

Instead of using the *old* and *new* subscripts, we will often simply think about a change from  $x$  to  $x + \Delta x$  (or from  $x$  to  $x + h$ ).

If  $f$  is a function and  $y = f(x)$ , then

$$\Delta y = y_{new} - y_{old} = f(x_{new}) - f(x_{old}) = f(x + \Delta x) - f(x).$$

Throughout the course, we will be interested in the relationship between  $\Delta x$  and  $\Delta y$ .



**Example 3** Let  $y = g(x) = x^3 - 2x$ . Simplify the expression for  $\Delta y$ .

$$\begin{aligned} \Delta y &= g(x + \Delta x) - g(x) = [(x + \Delta x)^3 - 2(x + \Delta x)] - [x^3 - 2x] \\ &= x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 2x - 2\Delta x - x^3 + 2x = 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - 2\Delta x \end{aligned}$$

**Example 4** Let  $y = f(x) = \frac{1}{x}$ . Simplify the expression for  $\Delta y$ . Then simplify the expression for  $\Delta y/\Delta x$ .

$$\begin{aligned} \Delta y &= f(x + \Delta x) - f(x) = \frac{1}{x + \Delta x} - \frac{1}{x} = \frac{x}{x(x + \Delta x)} - \frac{x + \Delta x}{x(x + \Delta x)} = \frac{-\Delta x}{x(x + \Delta x)} \\ \frac{\Delta y}{\Delta x} &= \frac{1}{\Delta x} \left( \frac{-\Delta x}{x(x + \Delta x)} \right) = \frac{-1}{x(x + \Delta x)} \end{aligned}$$

## Graphing without the calculator

Even though we will often make use of the graphing calculator, it is important to have basic graphing skills. When we need to have a “rough” graph of a basic function, we can normally get it very quickly without the calculator.

Here is a short list of the graphing skills that we are all expected to have:

- Know the graphs of basic functions such as  $y = mx + b$ ,  $y = x^n$ ,  $y = \sqrt{x}$ ,  $y = |x|$ ,  $y = \sin x$ ,  $y = \cos x$ , and  $y = \tan x$ .
- Know the general shape of the graph of a polynomial of degree  $n$ .
- Use  $x$ - and  $y$ -intercepts when graphing.

- Use vertical & horizontal translations and vertical & horizontal flips.
- Use symmetry.
  - A function is **even** if  $f(-x) = f(x)$ . The graph of an even function is symmetric about the  $y$ -axis.
  - A function is **odd** if  $f(-x) = -f(x)$ . The graph of an odd function is symmetric about the origin.
- Know a little bit about horizontal and vertical asymptotes.

**Example 4** Explain how the graph of  $y = (x + 1)^2 - 3$  can be obtained from the graph of  $y = x^2$ .

Start with the graph of  $y = x^2$  and shift it left 1 unit and down 3 units.

**Example 5** Sketch the graph of  $f(x) = (x - 4)(x + 2)$ .

*Solution omitted.*

**Example 6** Sketch the graph of  $g(x) = |\sin 2\pi x|$ .

*Solution omitted.*