Rolle's Theorem

Suppose f is continuous on $[a, b]$ and differentiable on (a, b). If $f(a) = f(b)$, then there exists a number c in (a, b) such that $f'(c) = 0$.

Mean Value Theorem

If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Increasing/decreasing functions — Definition

Suppose the function f is defined on an interval I.

- If for any two points x_1 and x_2 in I,

  $$x_1 < x_2 \quad \Longrightarrow \quad f(x_1) < f(x_2),$$

  then f is increasing on I.

- If for any two points x_1 and x_2 in I,

  $$x_1 < x_2 \quad \Longrightarrow \quad f(x_1) > f(x_2),$$

  then f is decreasing on I.

Increasing/decreasing functions — Theorem

If f is differentiable at each point of (a, b) and the derivative is positive at each point, then f is increasing on (a, b).

If f is differentiable at each point of (a, b) and the derivative is negative at each point, then f is decreasing on (a, b).