

Resolving ∞/∞

One approach to resolving the form ∞/∞ is to divide through by the highest power of x that appears in the denominator. Then re-evaluate the limit.

In other words, if x^r is the highest power of x in the denominator, then multiply both the numerator and denominator by $\frac{1}{x^r}$.

Definition of horizontal asymptote

The horizontal line $y = a$ is a horizontal asymptote of the graph of the function f if

$$\lim_{x \rightarrow \infty} f(x) = a \quad \text{or} \quad \lim_{x \rightarrow -\infty} f(x) = a.$$

Horizontal asymptotes of rational functions

Suppose R is a rational function (i.e. a quotient of two polynomials).

The asymptotes of the graph of R can be determined as follows...

- If the degree of the numerator is *less than* the degree of the denominator, then $y = 0$ is the horizontal asymptote of the graph of R .
- If the degree of the numerator is *equal to* the degree of the denominator, then $y = a/b$ is the horizontal asymptote of the graph of R , where a and b are the leading coefficients of the numerator and denominator, respectively.
- If the degree of the numerator is *greater than* the degree of the denominator, then the graph of R does not have a horizontal asymptote.