Definition of antiderivative

The function F is an antiderivative of the function f on the interval I if $F'(x) = f(x)$ for all x in I.

Theorem — Functions with identical derivatives

Two functions have the same derivative on the interval $[a, b]$ if and only if the functions differ by a constant.

That is, on the interval $[a, b]$,

$$F'(x) = G'(x) \quad \text{iff} \quad F(x) = G(x) + C.$$

Definition of indefinite integral

Suppose F is a function such that $F'(x) = f(x)$. The indefinite integral of f, denoted by $\int f(x)\,dx$, is the most general antiderivative of f, which includes an arbitrary *constant of integration*:

$$\int f(x)\,dx = F(x) + C.$$

The indefinite integral gives a generic description of every possible antiderivative.

Theorem — Properties of indefinite integrals

Suppose that antiderivatives of f and g exist.

- $\int k\,f(x)\,dx = k\int f(x)\,dx$ for any constant k

- $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$

- $\int [f(x) - g(x)]\,dx = \int f(x)\,dx - \int g(x)\,dx$