Definition of Riemann sum

Suppose f is defined on $[a, b]$. Partition $[a, b]$ into n subintervals:

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b,$$

and let Δx_k be the length of the kth subinterval. The sum

$$\sum_{k=1}^{n} f(c_k)\Delta x_k, \quad x_{k-1} \leq c_k \leq x_k$$

is called a Riemann sum for f with the given partition.

Definition of the definite integral

Suppose f is defined on $[a, b]$. Partition $[a, b]$ into n subintervals:

$$a = x_0 < x_1 < x_2 < \cdots < x_n = b,$$

and let Δx_k be the length of the kth subinterval. Further let $||\Delta|| = \max\{\Delta x_1, \Delta x_2, \ldots, \Delta x_n\}$. The *definite integral* of f on $[a, b]$ is given by

$$\int_a^b f(x)\,dx = \lim_{||\Delta|| \to 0} \sum_{k=1}^{n} f(c_k)\Delta x_k, \quad x_{k-1} \le c_k \le x_k,$$

provided the limit exists.

When the limit exists, f is said to be *Riemann integrable* on $[a, b]$. The function f is called the *integrand*.

Theorem — Continuous functions are integrable

If f is continuous on $[a, b]$, then f is Riemann integrable on $[a, b]$. That is, $\int_a^b f(x)\, dx$ exists.