Theorem — Area as a definite integral

Suppose f is a nonnegative, continuous function on $[a, b]$. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b f(x)\, dx.$$

Theorem — Basic properties of definite integrals

Suppose f and g are integrable on $[a, b]$ and $[b, c]$.

- $\int_a^a f(x)\,dx = 0$

- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

- $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ for any constant k

basic properties continued ...

- $\int_a^b [f(x) \pm g(x)]\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$

- $\int_a^b f(x)\, dx + \int_b^c f(x)\, dx = \int_a^c f(x)\, dx$

Theorem — Some inequalities involving definite integrals

Suppose f and g are integrable on $[a, b]$.

- If $f(x) \geq 0$ on $[a, b]$, then $\int_a^b f(x)\,dx \geq 0$.

- If $f(x) \geq g(x)$ on $[a, b]$, then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.

- If $m \leq f(x) \leq M$ on $[a, b]$, then

$$m \cdot (b - a) \leq \int_a^b f(x)\,dx \leq M \cdot (b - a).$$

Theorem — Even and odd functions in definite integrals

Suppose f is an even function (i.e. $f(-x) = f(x)$) and g is an odd function (i.e. $g(-x) = -g(x)$) on the interval $[-a, a]$.

- $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

- $\int_{-a}^{a} g(x)\,dx = 0$

Definition of average value

Suppose f is Riemann integrable on $[a,b]$. The average value of f on $[a,b]$ is given by

$$\text{Avg} = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

Mean Value Theorem for Integrals

If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

$$\int_a^b f(x)\,dx = f(c) \cdot (b - a).$$