**Simpson's rule over $n$ subintervals ($n$ even)**

Suppose $f$ is integrable on $[a, b]$. Partition $[a, b]$ into an even number $n$ of subintervals of equal length $h = \dfrac{b-a}{n}$:

$$a = x_0 < x_1 = a+h < x_2 = a+2h < \cdots < x_k = a+kh < \cdots < x_n = b.$$

Simpson's rule approximation for $\displaystyle\int_a^b f(x)\,dx$ is given by

$$\int_a^b f(x)\,dx \approx S,$$

where

$$S = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 4f(x_{n-1}) + f(x_n)\right].$$

**Theorem — Error in Simpson's rule**

Suppose $f$ has a continuous fourth derivative on $[a, b]$ and let $S$ be the Simpson's rule approximation for $\int_a^b f(x)\,dx$. The error in the approximation satisfies the inequality

$$\left| S - \int_a^b f(x)\,dx \right| \leq \frac{(b-a)^5}{180n^4} \max\{|f^{(4)}(x)| : a \leq x \leq b\}.$$