Definition of Continuity

A function is continuous at $x = c$ if the value the function is approaching as x approaches c is the value that the function actually attains at $x = c$.

In other words...
the function f is continuous at an interior point c of its domain if

$$\lim_{x \to c} f(x) = f(c).$$

If c is an interval endpoint, then the limit is replaced by a corresponding one-sided limit.

A function will fail to be continuous at a point if any one of these problems arises:

- the function is not defined at the point,
- the limit does not exist at the point, or
- the function value does not equal the limit.

Theorem — Properties of continuity

1. Polynomial functions are continuous everywhere.

2. Rational functions are continuous wherever they are defined.

3. The six basic trigonometric functions are continuous wherever they are defined.

4. Radical functions are continuous wherever they are defined.

5. Sums, differences, products, quotients, and compositions of continuous functions are continuous wherever they are defined.

Intermediate Value Theorem

Suppose f is continuous on the interval $[a, b]$. If k is any number between $f(a)$ and $f(b)$, then there must exist a number c in $[a, b]$ such that $f(c) = k$.