Section 2.1 - The Nature of Sets

A **set** is a collection of distinct objects. The objects that make up the set are called the **elements.**

We often give our sets short, convenient names.

Examples...

- Let *A* be the set of counting numbers strictly between 5 and 9.
- Let *M* be the set of students currently enrolled in Math 112-01.
- Let *L* be the set of letters of the word *ORANGE.*
- Let *Z* be the set of solutions of the equation $3x + 2 = 5$.

Sets must be **well defined** -- one must be able to determine objectively, without question, whether an object is an element or not.

The set of all pretty flowers is NOT well defined.

There are three common ways to define sets:

1. Verbal description - The set is defined entirely in words with no special notation. (See the examples above.)

2. Roster (or listing) notation - The elements of the set are listed between brackets. "..." may be used when the pattern is clear.
   a.) $A = \{6,7,8\}$
   b.) $L = \{O, R, A, N, G, E\}$
   c.) $Z = \{1\}$
   d.) $\mathbb{N} = \{1,2,3,4,5,6, \dots\}$ (This is the set of natural or counting numbers.)

3. Set builder notation - The elements are described mathematically with the following notation:

$$A = \{\, x \mid S(x) \,\}$$

We read this "*A is the set of all objects x such that S(x)*", where *S(x)* is a complete sentence stating the conditions that make *x* an element of the set.

a.) $A = \{x \mid x \text{ is a natural number and } 5 < x < 9\}$

b.) $Z = \{x \mid 3x + 2 = 5\,\}$

c.) $Z = \{y \mid 3y + 2 = 5\,\}$ (Same set! We do not need to use *x* as our variable!)

d.) $D = \{t \mid t \text{ is a positive number that satisfies } t + 1 = 0\}$

Notice that D has no elements! It is the **empty set**.

$$D = \{\ \} = \emptyset$$

The symbol "∈" means "*is an element of.*"

For example,

- $56 \in \mathbb{N}$
- $100 \in \{2,4,6,8,10,12, \dots\}$
- $q \in \{a, b, c, d, \dots, x, y, z\}$
- $-5 \notin \mathbb{N}$
- $173 \notin \{2,4,6,8,10,12, \dots\}$
- $a \notin \{d, o, g\}$

Two sets are **equal** if they have exactly the same elements.

Important note...
Based on the definition of set equality, if sets are unequal, there must be elements that they do not have in common.

- $\{1,2,3\} = \{3,1,2\}$
- $\{1,2,3\} = \{1,2,2,3,3,3\}$
- $\{1,2,3\} \neq \{1,2,3,4\}$
- $\{1,2,3, \dots ,98,99,100\} \neq \mathbb{N}$
- $\{a, b, c\} = \{c, a, b, b, a\}$
- $\{a, b, c, x, y, z\} \neq \{a, b, c, \dots , x, y, z\}$

The number of elements of a set is called the **cardinal number** or the **cardinality** of the set.

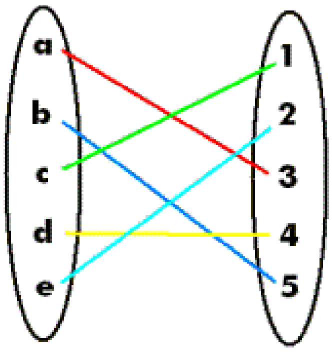
The cardinality of the set A is denoted by $n(A)$.

- $n(Z) = 1$ (Refer back to the set Z defined above.)
- $n(\{0,1,2,3,4,5,6,7,8,9\}) = 10$
- $n(\{a, b, c, \dots, x, y, z\}) = 26$
- $n(\mathbb{N}) = \infty$
- $n(\emptyset) = 0$

If two sets are equal, what can you say about their cardinal numbers?

A **one-to-one correspondence** is a pairing of two sets such that each element of one set is paired with exactly one element of the other set and vice versa.

The following diagram shows one of many one-to-one correspondences between $\{a, b, c, d, e\}$ and $\{1,2,3,4,5\}$.



Two sets A and B are said to be **equivalent**, written $A \cong B$, if there is a one-to-one correspondence between them.

- $\{a, b, c, d, e\} \cong \{1,2,3,4,5\}$
- $\{1,3,5,7, \dots\} \cong \{2,4,6,8, \dots\}$
- $\{\emptyset\} \cong \{5\}$
- $\{10,20,30,40, \dots\} \cong \mathbb{N}$

Theorem
Two finite sets are equivalent if and only if they have the same cardinal number.

Examples...

- Give an example of a set that is equivalent to, but not equal to, $\{a, b, c, \dots x, y, z\}$.

- Give an example of a single set A such that $n(A) = 4$, $g \in A$, and $17 \in A$.

- Let $A = \{x | x \in \mathbb{N}, 4 \leq x < 8\}$ and let B be the set of letters of the word *MISSISSIPPI*. Is it true that $A \cong B$? If so, show a one-to-one correspondence.