Section 2.2 - Subsets and Set Operations

If every element of the set A is also an element of the set B, then we say that A is a **subset** of B.

We use the notation $A \subseteq B$ to write "A is a subset of B."

Examples….

Let $A = \{1,2,3\}$, $B = \{1,2,3,4\}$, and $C = \{2,3,4\}$.

- $A \subseteq B, \quad C \subseteq B$
- $B \nsubseteq A$
- $\emptyset \subseteq A, \quad \emptyset \subseteq B, \quad \emptyset \subseteq C$
- $\{1\} \subseteq A$
- $1 \in A$
- $1 \nsubseteq A$
- $\{1\} \notin A$

Examples…

- List all subsets of $\{a\}$.

- List all subsets of $\{1,2\}$.

- List all subsets of $\{x, y, z\}$.

Theorem

If a finite set has k elements, then it has 2^k subsets.

A subset of A is called **proper** if it is unequal to A. We use the notation $A \subset B$ to write "A is a proper subset of B."

Every set we study has elements that are taken from a larger pool of potential elements.

In a particular situation, the **universal set**, U, is the set of all potential elements under consideration. For a particular situation, U is the universe to us!

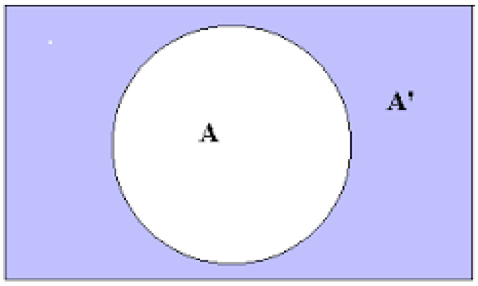
Examples…

- If rolling a six-sided die, $U = \{1,2,3,4,5,6\}$.
- Thinking about 1-digit numbers, $U = \{0,1,2,3,4,5,6,7,8,9\}$.
- Thinking about letters, $U = \{a, b, c, \dots, x, y, z\}$.

The **complement** of A, written A', is the set of all elements in U, but not in A.

$$A' = \{x \mid x \in U \text{ and } x \notin A\}$$

The following Venn diagram illustrates the complement of A (shaded).



Examples…

Let $U = \{0,1,2,3,4,5,6,7,8,9\}$ and let $E = \{0,2,4,6,8\}$.

- $E' = \{1,3,5,7,9\}$
- $\{1,2,3,4,5,6,7,8\}' = \{0,9\}$
- $U' = \emptyset$
- $\emptyset' = U$

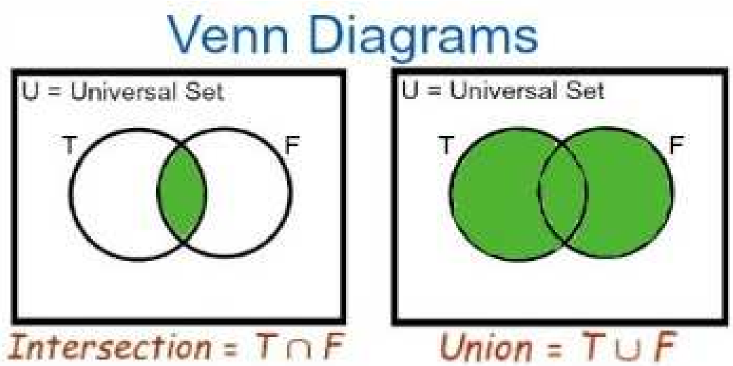
The **intersection** of the two sets A and B is the set of all elements common to both sets.

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

The **union** of the two sets A and B is the set of all elements that are in one, the other, or both.

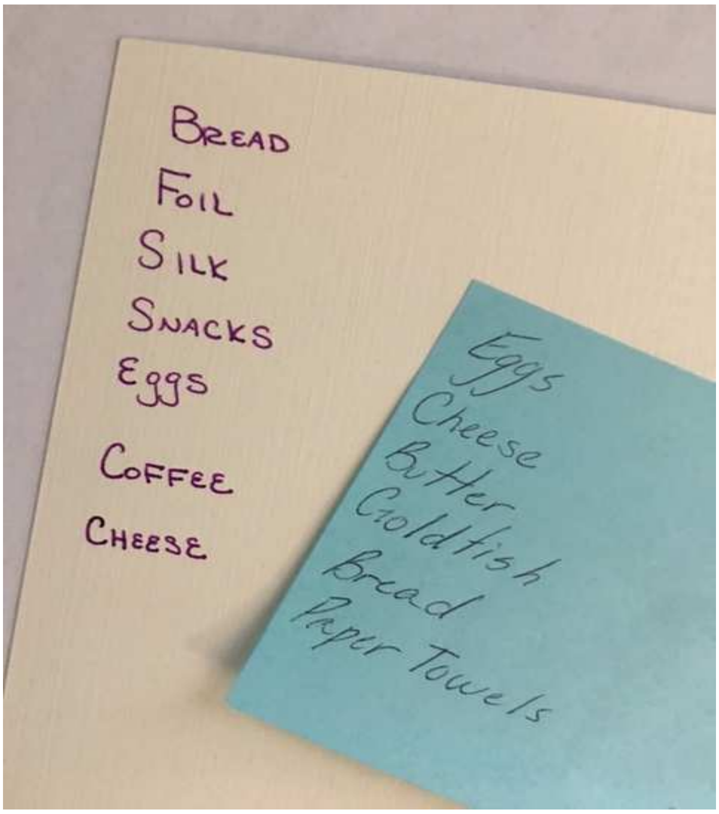
$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

The union is the joining of sets, while the intersection is the overlap. The following Venn diagrams illustrate these ideas.



Example...

Two shopping lists are shown below. What are the union and intersection of the sets of items?



Example...

Let $A = \{m, i, s, p\}$ and let $B = \{a, l, s, k\}$. Determine each set.

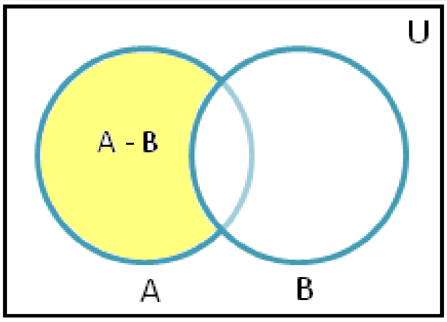
- $A \cup B$
- $A \cap B$
- $A \cap \emptyset$
- $B \cup \emptyset$
- $A' \cap B$

The **difference** of the set A and the set B is the set of all elements in A that are *not* in B.

$$A - B = \{ x \mid x \in A \text{ and } x \notin B \}$$

Or

$$A - B = \{ x \mid x \in A \text{ and } x \in B' \}$$



Difference of two sets
A - B

Examples…

Let $A = \{1,2,3,4,5,6,7,8\}$, $B = \{2,4,6,8\}$, and $C = \{2,3,5\}$.

- $A - B = \{1,3,5,7\}$
- $B - A = \emptyset$
- $B - C = \{4,6,8\}$
- $C - B = \{3,5\}$

The **Cartesian product** of sets A and B is the set of all ordered pairs in which the first coordinate is an element of A and the second coordinate is an element of B.

$$A \times B = \{ (x, y) \mid x \in A \text{ and } y \in B \}$$

Examples…

Let $A = \{a, b, c\}$ and $B = \{1,2\}$.

- $A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)\}$
- $B \times A = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$

Let $X = \{\text{red, blue, black, yellow}\}$ and $Y = \{\text{ford, chevy}\}$.

- List some elements of $X \times Y$.

- Determine $n(X \times Y)$.