1. Evaluate the following limit.

$$\lim_{t \to 2} \frac{\sqrt{t^2 - 3}}{t^2 - t + 3}$$

(a) 0

(b) 1/5

(c) 0/0

(d) The limit does not exist.

Solution

1

2. Evaluate the following limit.

$$\lim_{x \to 7} \frac{2x - 14}{x^2 - 6x - 7}$$

(a) 0/0

(b) 0

(c) 1/4

(d) The limit does not exist.

Solution

3. Evaluate the following limit.

$$\lim_{x \to 0} \frac{\sqrt{x + 13} - \sqrt{13}}{x}$$

(a) $1/(2\sqrt{13})$

(b) $0/0$

(c) 0

(d) The limit does not exist.

Solution

4. Evaluate the following limit.

$$\lim_{x \to 0} \frac{3x}{\tan 5x}$$

(a) 0/0

(b) 3/5

(c) 5/3

(d) The limit does not exist.

Solution

5. Evaluate the following limit: $\displaystyle\lim_{x \to 1} \dfrac{\dfrac{1}{2x+1} - \dfrac{1}{3}}{x-1}$

(a) $-2/9$

(b) $0/0$

(c) $1/3$

(d) The limit does not exist.

Solution

Problem 1 — The answer is (b).

The limit can be determined by direct substitution.

$$\lim_{t \to 2} \frac{\sqrt{t^2 - 3}}{t^2 - t + 3} = \frac{\sqrt{2^2 - 3}}{2^2 - 2 + 3} = \frac{1}{5}$$

Problem 2 — The answer is (c).

Direct substitution yields the indeterminate form 0/0. We factor and cancel...

$$\lim_{x \to 7} \frac{2x - 14}{x^2 - 6x - 7} = \lim_{x \to 7} \frac{2\cancel{(x - 7)}}{\cancel{(x - 7)}(x + 1)} = \lim_{x \to 7} \frac{2}{x + 1} = \frac{2}{8} = \frac{1}{4}$$

Problem 3 — The answer is (a).

Direct substitution yields the indeterminate form 0/0. We multiply by the conjugate, only expanding the numerator.

$$\lim_{x \to 0} \frac{\sqrt{x+13} - \sqrt{13}}{x} \cdot \frac{\sqrt{x+13} + \sqrt{13}}{\sqrt{x+13} + \sqrt{13}} =$$

$$\lim_{x \to 0} \frac{(x+13) - 13}{x(\sqrt{x+13} + \sqrt{13})} = \lim_{x \to 0} \frac{\not{x}}{\not{x}(\sqrt{x+13} + \sqrt{13})} = \frac{1}{\sqrt{13} + \sqrt{13}}$$

Problem 4 — The answer is (b).

Direct substitution yields the indeterminate form 0/0. We'll use the fact that

$$\lim_{u \to 0} \frac{\sin u}{u} = \lim_{u \to 0} \frac{u}{\sin u} = 1$$

$$\lim_{x \to 0} \frac{3x}{\tan 5x} = \lim_{x \to 0} \left(\frac{3}{5} \cdot \frac{\cos 5x}{1} \cdot \frac{5x}{\sin 5x} \right) = \frac{3}{5} \cdot 1 \cdot 1$$

Problem 5 — The answer is (a).

Direct substitution yields the indeterminate form 0/0. We'll simplify by carrying out the subtraction in the numerator.

$$\lim_{x \to 1} \frac{\dfrac{1}{2x+1} - \dfrac{1}{3}}{x-1} = \lim_{x \to 1} \frac{\dfrac{3-(2x+1)}{3(2x+1)}}{x-1} = \lim_{x \to 1} \frac{-2x+2}{3(2x+1)(x-1)}$$

$$= \lim_{x \to 1} \frac{-2\cancel{(x-1)}}{3(2x+1)\cancel{(x-1)}} = -\frac{2}{9}$$