Definition of absolute extrema

Suppose f is defined on a set D containing the number c.

- $f(c)$ is the absolute maximum value of f on D if $f(c) \geq f(x)$ for all x in D.

- $f(c)$ is the absolute minimum value of f on D if $f(c) \leq f(x)$ for all x in D.

Extreme Value Theorem

If f is continuous on the closed and bounded interval $[a, b]$, then f attains both an absolute minimum value and an absolute maximum value on $[a, b]$.

Definition of relative extrema

Suppose f is a function with domain D.

- If, inside D, there is an open interval containing c on which $f(c)$ is an absolute maximum, then $f(c)$ is a relative maximum.

- If, inside D, there is an open interval containing c on which $f(c)$ is an absolute minimum, then $f(c)$ is a relative minimum.

Definition of critical number

If c is an interior point in the domain of f at which $f'(c) = 0$ or $f'(c)$ does not exist, then c is called a critical number (or critical point) of f.

Theorem — Relative extrema and critical numbers

If f takes on an extreme value at a domain interior point c, then c is a critical number.

Finding absolute extrema

To find the absolute extreme values of f on $[a, b]$:

1. Find all critical numbers in (a, b).

2. Evaluate f at all critical numbers and at the domain endpoints, a and b.

3. Of the function values computed in step 2, the greatest is the absolute maximum, and the least is the absolute minimum.