Section 4.3 - Conditional Probability

In some situations, probabilities of events change when more information is obtained. For example...

What is the probability that you will carry an umbrella on any given day?

What is the probability that you will carry an umbrella on any given day, if you know it is raining that day?

The second probability is called a **conditional probability**.

Suppose *A* and *B* are events, $P(A|B)$ represents the probability of event *A* occurring after it is assumed that event *B* has already occurred. $P(A|B)$ is "the probability of *A* given *B*."

For any events A and B,

$$P(A \cap B) = P(A|B) \times P(B) = P(B|A) \times P(A).$$

Two events *A* and *B* are **independent** if the occurrence of one does not affect the probability of the other.

For example...

Select two letters at random from the word MISSISSIPPI. Let *A* be the event of selecting the letter S and let *B* be the event of selecting the letter P.

If the selections are made with replacement, *A* and *B* are independent.

If the selections are made without replacement, *A* and *B* are dependent.

If the size of a sample is no more than 5% of the population size, we will sometimes approximate probabilities by treating events as being independent, even if selections are made without replacement.

Recall...

For any events A and B,

$$P(A \cap B) = P(A|B) \times P(B) = P(B|A) \times P(A).$$

This formula can be rewritten to say

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Important ideas to keep in mind:

- In general, $P(A|B) \neq P(B|A)$.

- *A* and *B* are independent if and only if $P(A|B) = P(A)$ **and** $P(B|A) = P(B)$.