Section 4.4 - Counting

The Fundamental Principle of Counting

If there are n ways to do a first task and m ways to do a second task, then there are $n \times m$ ways to do the combination of tasks.

This generalizes to more tasks, and it follows that...

A collection of n different items can be arranged in $n!$ different ways (counting different orders as different ways).

Permutations (when items are different)

If the following requirements are met:

1. There are n different items available.
2. We select r of the n items (without replacement).
3. We consider rearrangements of the same items to be different (i.e. order matters).

The number of permutations of r items selected from n different items is

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

Permutations (when some items are identical)

If the following requirements are met:

1. There are n items available, and n_1 of them are alike, n_2 of them are alike,..., n_k of them are alike.
2. We select all of the n items (without replacement).
3. We consider rearrangements of distinct items to be different (i.e. order of different items matters).

The number of permutations of all n items selected without replacement is

$$\frac{n!}{n_1!\,n_2!\,\ldots\,n_k!}$$

Combinations

If the following requirements are met:

1. There are n different items available.
2. We select r of the n items (without replacement).
3. We consider rearrangements of the same items to be the same (i.e. order does not matter).

The number of combinations of r items selected from n different items is

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$