Section 5.2 - Binomial Distributions

A **binomial probability distribution** results from a procedure that meets all of the following:

1. The procedure has a fixed number of trials.
2. The trials must be independent.
3. Each trial must have outcomes that can be classified in exactly two categories: *success* or *failure*.
4. The probability of a success remains the same for all trials.

If a procedure satisfies the four requirements, the distribution of the random variable x, where x is the number of successes, is called a **binomial distribution**.

In some cases, the trials are not technically independent, but may be treated as such. Recall our earlier guideline:

If the size of a sample is no more than 5% of the population size, we will sometimes approximate probabilities by treating events as being independent, even if selections are made without replacement.

Notation for Binomial Distributions

- Probability of success = p (Single trial)
- Probability of failure = $q = 1 - p$ (Single trial)
- Number of trials = n
- Number of successes in n trials = x
- Probability of x successes in n trials = $P(x)$

By using the appropriate counting techniques, it can be shown that

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

Since the formulas from above for mean, variance, and standard deviation apply for any discrete probability distributions, they certainly apply for binomial distributions.

- Mean for a probability distribution (Expected Value)

$$\mu = \sum [x \times P(x)]$$

- Variance for a probability distribution

$$\sigma^2 = \sum [(x - \mu)^2 \times P(x)]$$

or

$$\sigma^2 = \sum [x^2 \times P(x)] - \mu^2$$

- Standard deviation for a probability distribution

$$\sigma = \sqrt{\sigma^2}$$

However, for binomial distributions, these formulas can be greatly simplified:

- $\mu = np$
- $\sigma^2 = npq$
- $\sigma = \sqrt{npq}$

Recall, once again, the rules of thumb for identifying the minimum and maximum "usual" values:

minimum "usual" value = $\mu - 2\sigma$
maximum "usual" value = $\mu + 2\sigma$