Sections 6.1 & 6.2 - Uniform and Normal Distributions

If a continuous random variable has a distribution whose graph is a symmetric, bell-shaped curve described by an equation of the form

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma \cdot \sqrt{2\pi}},$$

we say that the random variable has a **normal distribution**.

Notice that the normal distribution is determined by two parameters: the mean and the standard deviation.

The graph of a continuous probability distribution is called a **density curve**.

Every density curve must satisfy the following requirements:

1. The total area under the curve must equal 1.
2. The y-coordinate of every point on the curve must be greater than or equal to zero. (The curve cannot fall below the x-axis.

Because the total area under a density curve is equal to 1, there is a correspondence between area and probability.

Uniform Distribution

A continuous random variable has a **uniform distribution** if its values are spread evenly over the range of possible values (i.e. all possible values are equally likely). The graph of a uniform distribution is a horizontal line segment.

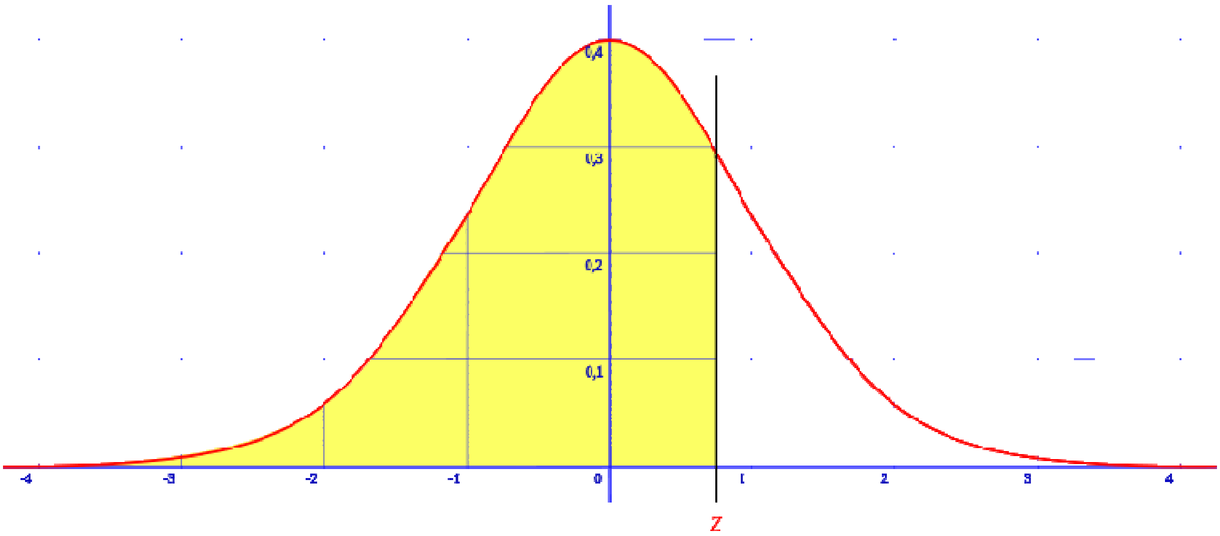
Standard Normal Distribution

The **standard normal distribution** is the normal distribution with mean zero ($\mu = 0$) and standard deviation one ($\sigma = 1$).

If x is a random variable in a normal distribution, then $z = \frac{x-\mu}{\sigma}$ is the corresponding random variable in the standard normal distribution.

Notice that if $\mu = 0$ and $\sigma = 1$, then $z = x$, and your random variable is already *normalized*.

For any given z-score, the probability of a score less than or equal to z is the area under the standard normal curve to the left of z.



Computing the area under the curve is a calculus problem. We'll find area by using our calculators or a table.

For example...

- $P(z < 1) = 0.8413$
- $P(z < 2.09) = 0.9817$
- $P(z < -1.24) = 0.1075$
- $P(-1.23 < z < 1.98) = 0.8668$
- $P(z > 1.3) = 1 - P(z < 1.3) = 0.09680$

For each of these, shade the appropriate region under the standard normal curve.

Examples

1. Temperatures are normally distributed with mean 0 and standard deviation 1. In a sample of 500 temperature measurements, about how many lie between -1.38 and 0.87?
2. Birth weights in Norway are normally distributed with mean 3570 grams and standard deviation 500 grams. What percent of newborns have weight less than 3100 grams? In a year, a certain hospital delivers 600 babies. How many weigh less than 2900 grams?
3. Men's heights are normally distributed with mean 69.0 in and standard deviation 2.8 in. What percent of men are shorter than 65 in or taller than 72 in?

In some problems we'd like to find the z-score associated with a certain probability. This is the inverse of the type of problem we did above.

Example

Refer to Example 3 above. What height should a doorway be so that 95% of men can walk through without bending down?