Section 6.4 - Central Limit Theorems

**Central Limit Theorem**

1. For a population with any distribution, if $n > 30$, then the sample means have a distribution that can be approximated by a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.
2. If $n \leq 30$ and the original population has a normal distribution, then the sample means have a normal distribution with mean μ and standard deviation $\sigma/\sqrt{n}$.

*There are exceptions to the rule in (1). Some populations may require a larger sample size.*

Some things to keep in mind...

- As the sample size increases, the distribution of sample means gets closer and closer to a normal distribution.
- The mean of sample means is the same as the mean of the original population.
- As the sample size increases, the distribution of sample means gets narrower (because the standard deviation of sample means gets smaller).

## Notation for the Sampling Distribution of the Mean

If all possible random samples of size $n$ are selected from a population with mean μ and standard deviation $\sigma$, then the mean of the sample means is denoted by $\mu_{\bar{x}}$. Note that $\mu_{\bar{x}} = \mu$.

The standard deviation of sample means is denoted by $\sigma_{\bar{x}}$. Note that $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. In this context, $\sigma_{\bar{x}}$ is often called the **standard error of the mean**.

**Central Limit Theorem for Proportions**

Let $\hat{p}$ be the sample proportion for a sample of size $n$ and population proportion $p$ (with $q = 1 - p$). If

$$np \geq 10 \text{ and } nq \geq 10$$

then the sampling distribution of proportions for samples of size $n$ is approximately normal with mean $\mu_{\hat{p}} = p$ and

$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$.

**Example**

The new Endeavor SUV has been recalled because 5% of the cars experience brake failure. A random sample of 200 cars is obtained. What is the probability that the proportion of defective cars in the sample is less than 4%?