Section 7.2 - Confidence Intervals for the Mean (Part 2)

To construct a confidence interval for the population mean μ:

Assuming...

1. The sample is a simple random sample.
2. The value of the population standard deviation σ is known.
3. Either the population is normally distributed or $n > 30$.

The confidence interval is

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},$$

where $E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ is the margin of error.

The correct interpretations of this confidence interval are similar to those given in Section 7.1.

Sample size required to estimate a population mean:

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2$$

If σ is not known, there are several ways to approximate it.

- Using the range rule of thumb, $\sigma \approx$ range/4.
- Collect sample values and use $\sigma \approx s$.
- Estimate σ using the results of another study or survey.

Section 7.2 (Part 1)

When estimating a population mean when σ is not known, we use the Student's t-Distribution.

If a population has a normal distribution, then the distribution of

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

is a **Student t-distribution** for all samples of size n.

To construct a confidence interval for the population mean μ, when σ is not known:

Assuming...

1. The sample is a simple random sample.
2. Either the population is normally distributed or $n > 30$.

The confidence interval is

$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}},$$

where $E = t_{\alpha/2} \cdot \frac{s}{\sqrt{n}}$ is the margin of error and $t_{\alpha/2}$ is obtained using $n - 1$ degrees of freedom.

Important Properties of Student's t-Distribution:

1. The distribution is different for different sample sizes.
2. The distribution has the same general symmetric bell shape as the standard normal distribution, but it reflects the greater variability that is expected with small samples.
3. The distribution has a mean of $t = 0$.
4. The standard deviation of the distribution varies with sample size, but it is greater than 1.
5. As the sample size gets larger, the distribution gets closer to the standard normal distribution.