<u>Section 7.3 - Confidence Intervals for the Variance</u>

When estimating a population standard deviation σ or population variance σ^2, we use a χ^2 (chi-square) distribution.

Warning: The chi-square distribution is not a symmetric distribution, so confidence interval estimates for the standard deviation and variance are not centered on the point estimate.

If a population has a normal distribution, then the distribution of

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

is a **chi-square distribution** for all samples of size n.

To construct a confidence interval for the population variance σ^2:

Assuming...

1. The sample is a simple random sample.
2. The population is normally distributed.

The confidence interval is

$$\frac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_L^2},$$

where χ_R^2 and χ_L^2 are the right- and left-tailed critical numbers obtained using $n-1$ degrees of freedom.

If a confidence interval estimate of σ is required, it is obtained by taking square roots of the bounds given above.

Important Properties of Chi-Square Distribution:

1. The distribution is not symmetric. However, as the number of degrees of freedom increases, the distribution becomes more symmetric.
2. The values of chi-square can be zero or positive, but they cannot be negative.
3. As the sample size gets larger, the distribution gets closer to a normal distribution.