Math 112 - Test 2
October 13, 2016

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Which of these sentences are statements? Circle all that apply.

   (a) Bill Gates is the founder of Microsoft.
   (b) Who will be elected president?
   (c) $12 + 8 = 13$
   (d) Leonard Euler was born in 1707.
   (e) Be cool!

2. (5 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

   (a) All dogs are mammals.

   CONDITIONAL

   (b) Sara and Brian went hiking in the mountains.

   CONJUCTION

   (c) I'm going to pass my math class if and only if I do my homework.

   BICONDITIONAL

   (d) When your battery dies, you should charge your phone.

   CONDITIONAL

   (e) Either he makes the shot, or he loses the game.

   DISJUNCTION

3. (3 points) Explain why the sentence "This statement is false" is not a statement.

   IT TRUTH VALUE CANNOT BE DETERMINED.

   IF IT IS FALSE, THEN IT MUST BE TRUE.

   IF IT IS TRUE, THEN IT MUST BE FALSE.

4. (6 points) Write the negation of each statement in a correct sentence.

   (a) Steve likes to drink iced tea.

   STEVE DOES NOT LIKE TO DRINK ICED TEA.

   (b) Everyone in the class was bored by the professor's lecture.

   SOME IN CLASS WERE NOT BORED ...

   (c) Some people watch *Game of Thrones*.

   ALL PEOPLE DO NOT WATCH GAME OF THRONES.
   -OR-
   NO ONE WATCHES GAME OF THRONES.

5. (6 points) Let p = "The dog is a poodle" and let q = "The dog bites." Write each statement in words.

   (a) $q \longrightarrow \sim p$ IF THE DOG BITES, THEN IT IS NOT A POODLE.

   (b) $\sim(q \wedge p)$ THE DOG IS NOT A BITING POODLE.

   (c) $p \longrightarrow (p \vee q)$

   IF THE DOG IS A POODLE, THEN IT IS A POODLE OR IT BITES.

6. (6 points) Refer to the statements p and q from the problem directly above. Write each statement in symbolic form.

   (a) The dog bites, but it is not a poodle.

   $q \wedge \sim p$

   (b) The dog bites if it is a poodle.

   $p \longrightarrow q$

   (c) The dog bites if and only if it is not a poodle.

   $q \longleftrightarrow \sim p$

7. (6 points) Construct the truth table for $(\sim q \wedge p) \longrightarrow \sim p$.

p	q	$\sim q$	$\sim q \wedge p$	$\sim p$	$\sim q \wedge p \rightarrow \sim p$
T	T	F	F	F	T
T	F	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T

8. (2 points) If p is false, what is the truth value of $\sim(\sim(\sim p))$?

$\sim p$ is T

$\sim(\sim p)$ is F

$\sim(\sim(\sim p))$ is T

TRUE

9. (4 points) Suppose that the following conditional statement is true:

If Sarah gets a good lawyer, then she will not go to jail.

(p: *Sarah gets a good lawyer*; q: *she will not go to jail*)

What valid conclusion can we reach if Sarah goes to jail? ($\sim q$) Use a common form of argument to briefly explain how you know your conclusion is valid.

$p \rightarrow q \equiv \sim q \rightarrow \sim p$

CONTRAPOSITIVE!

SARAH DID NOT GET A GOOD LAWYER.

$\sim p$

10. (5 points) Consider the following conditional statement:

If Sam takes his medicine, then he will get well.

(a) State the inverse.

IF SAM DOES NOT TAKE HIS MEDICINE, THEN HE WILL NOT GET WELL.

(b) State the contrapositive.

IF SAM DOES NOT GET WELL, THEN HE DID NOT TAKE HIS MEDICINE.

(c) State the converse.

IF SAM GETS WELL, THEN HE TOOK HIS MEDICINE.

(d) Of the three, which is equivalent to the original statement?

Inverse (Contrapositive) Converse

11. (2 points) When is the implication $p \longleftrightarrow q$ false?

WHEN p & q HAVE OPPOSITE TRUTH VALUES.

12. (6 points) Use truth tables to show that the statement $p \longrightarrow q$ is logically equivalent to $\sim p \vee q$.

p	q	~p	~p v q	p → q
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

IDENTICAL TRUTH TABLES.

13. (4 points) Use DeMorgan's Laws to write a logically equivalent statement.

(a) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

(b) $\sim(q \wedge r) \equiv \sim q \vee \sim r$

14. (4 points) Write the negation of the statement "The coffee is a latte or an espresso."

THE COFFEE IS NOT A LATTE AND NOT AN ESPRESSO.

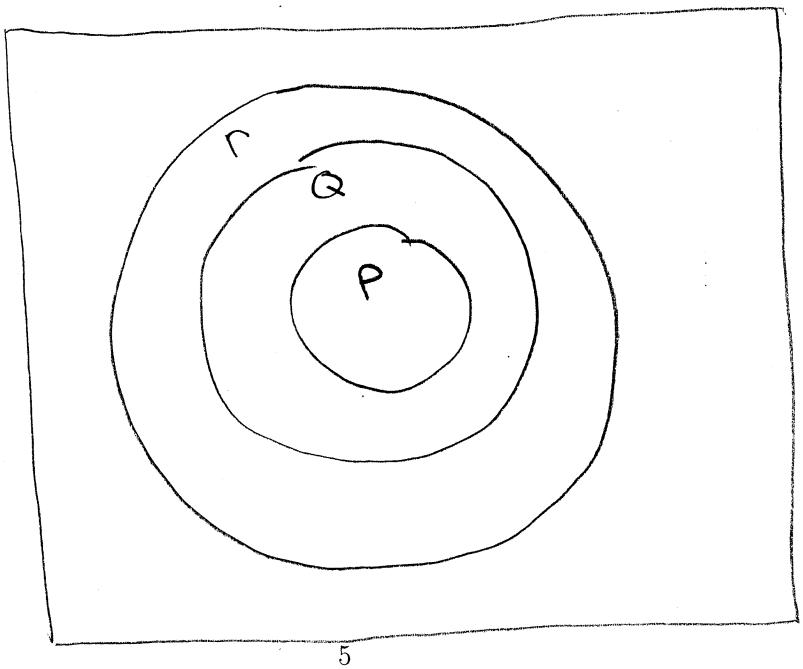
15. (6 points) Use Euler circles to determine the validity of the following argument.

$$\begin{array}{l} p \longrightarrow q \\ q \longrightarrow r \\ \hline \therefore\ p \longrightarrow r \end{array}$$

ALL P ARE IN R.

VALID!

IN FACT, THIS IS A TRANSITIVE ARGUMENT.



16. (9 points) Consider the following argument in symbolic form.

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array}$$

(a) Use the truth table method to determine the validity of the argument.

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge (\sim p)$	$(p \vee q) \wedge (\sim p) \rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

↑ TAUTOLOGY!

ARGUMENT IS VALID.

(b) Is the argument a common form? If so, use your knowledge of common forms to explain the validity.

THIS IS A COMMON FORM --- DISJUNCTIVE SYLLOGISM.

IT IS THE "ONE OR ANOTHER AND NOT ONE" ARGUMENT.

IT IS A VALID COMMON FORM.

17. (5 points) Give a symbolic example of a common-form argument that is invalid. Explain how you know it is invalid.

$$\begin{array}{l} p \rightarrow q \\ \sim p \\ \hline \therefore \sim q \end{array}$$

THIS IS THE FALLACY OF THE INVERSE.

18. (8 points) Write the argument in symbolic form. Then use a common-form argument to determine its validity. Explain your reasoning.

Premise 1: George will pass the class (q) if he gets an A on the last test (p).

Premise 2: If George gets an A on the last test (p), then he will not have to pay for the class (r).

Conclusion: If George will pass the class, then George will not have to pay for the class.

$p \to q$

$p \to r$

$\therefore q \to r$

THIS IS NOT VALID. q HAS NO CONNECTION TO r BY THE PREMISES.

IT ALMOST LOOKS TRANSITIVE, BUT NOT.

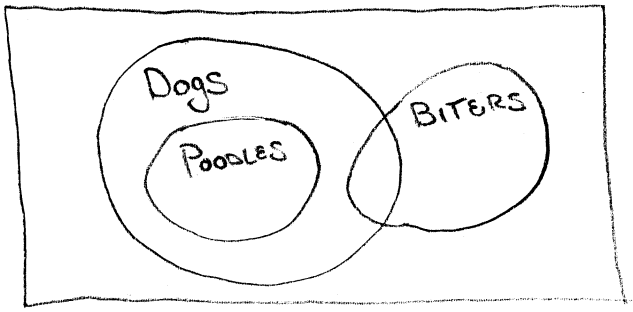
19. (8 points) Consider the argument shown below.

Premise 1: Some dogs bite.

Premise 2: All poodles are dogs.

Conclusion: Some poodles bite.

(a) Sketch an Euler circle diagram that shows the argument is invalid.



(b) Even though the argument is invalid, some Euler circle diagrams satisfy the two premises and the conclusion. Draw such a diagram.