

**Math 112 - Test 2**  
March 8, 2017

Name key  
Score \_\_\_\_\_

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

(a) Either he passes the test, or he fails the course.

DISJUNCTION

(b) A number is even if and only if it is divisible by 2.

BICONDITIONAL

(c) You should get a job if you need money.

CONDITIONAL

(d) It is going to rain or snow.

DISJUNCTION

(e) She listened to her iPod and did her homework.

CONJUNCTION

2. (3 points) Give an example of a sentence that is not a statement and explain why it is not a statement.

HAVE A GOOD DAY.

THIS SENTENCE CANNOT HAVE A TRUTH VALUE.

IT IS A DIRECTIVE RATHER THAN A SENTENCE THAT  
CAN BE TRUE OR FALSE.

3. (6 points) Which of these sentences are statements? Circle all that apply.

(a) Please do not sleep during class.

☒ (b) David Pumpkins is taking Math 112.

(c) Do some push-ups.

☒ (d)  $9 - 3 = 2$

☒ (e) Alfred Tarski was a famous mathematician.

(f) Well, hello there.

4. (8 points) Write the negation of each statement in a correct sentence.

(a) I do not like to eat seafood.

I LIKE TO EAT SEAFOOD.

(b) Every happy dog wags its tail.

SOME HAPPY DOGS DO NOT WAG THEIR TAILS.

(c) Some students write with crayons.

NO STUDENTS WRITE WITH CRAYONS.

(d) No one gets out alive.

SOME ONE GETS OUT ALIVE.

5. (6 points) Let  $p$  = "The train arrives on time" and let  $q$  = "It is snowing." Write each statement in words.

(a)  $\sim q \rightarrow p$  IF IT IS NOT SNOWING, THEN THE TRAIN ARRIVES ON TIME.

(b)  $\sim(q \vee p)$  IT IS NOT SNOWING AND THE TRAIN DOES NOT ARRIVE ON TIME.  
 $\sim q \wedge \sim p$

(c)  $q \leftrightarrow (p \wedge q)$

IT IS SNOWING IF AND ONLY IF IT IS SNOWING AND THE TRAIN ARRIVES ON TIME.

6. (6 points) Refer to the statements  $p$  and  $q$  from the problem directly above. Write each statement in symbolic form.

(a) The train arrives on time, or it is not snowing.

$p \vee \sim q$

(b) The train does not arrive on time whenever it is snowing.

$q \rightarrow \sim p$

(c) It is snowing, but the train arrives on time.

$q \wedge p$

7. (6 points) Construct the truth table for  $(p \vee q) \rightarrow \sim p$ .

$p$	$q$	$p \vee q$	$\sim p$	$(p \vee q) \rightarrow \sim p$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

8. (8 points) Construct the truth table for  $p \wedge (q \vee \sim r)$ .

$p$	$q$	$r$	$\sim r$	$q \vee \sim r$	$p \wedge (q \vee \sim r)$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	T	T	F

9. (2 points) Suppose  $p$  is false and  $q$  is true. What is the truth value of  $p \leftrightarrow q$ ?

$F \leftrightarrow T$  IS FALSE.

$P \leftrightarrow q$  IS ONLY  
TRUE WHEN BOTH  
HAVE SAME  
TRUTH  
VALUES.

10. (6 points) Let  $p =$  "Today is Saturday" and let  $q =$  "Some dogs bite." Determine the truth value of each of the following statements.

- (a)  $p \vee \sim q$

$$F \vee \sim T = F \vee F = \boxed{\text{FALSE}}$$

- (b)  $p \rightarrow q$

$$F \rightarrow T = \boxed{\text{True}}$$

- (c)  $(p \wedge q) \rightarrow \sim p$

$$\underbrace{(F \wedge T)}_{F} \rightarrow \sim F$$

$$F \rightarrow T = \boxed{\text{True}}$$

11. (6 points) Write the truth table for  $p \leftrightarrow q$ .

$p$	$q$	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

12. (2 points) True or False: The biconditional statement  $p \leftrightarrow q$  means the same as  $(p \rightarrow q) \vee (q \rightarrow p)$ ?

FALSE, IT MEANS  $(p \rightarrow q) \wedge (q \rightarrow p)$

13. (8 points) Consider the following conditional statement:

*If Sarah wins the award, then she will get a good job.*

- (a) State the inverse.

IF SARAH DOES NOT WIN THE AWARD, THEN SHE WILL NOT GET A GOOD JOB.

- (b) State the contrapositive.

IF SHE DOES NOT GET A GOOD JOB, THEN SARAH DOES NOT WIN THE AWARD.

- (c) State the converse.

IF SHE GETS A GOOD JOB, THEN SARAH WINS THE AWARD.

- (d) Of the three, which is logically equivalent to the original statement?

Inverse

Contrapositive

Converse

14. (2 points) Without actually constructing it, determine how many rows the truth table for  $(p \vee q) \wedge (r \wedge s)$  would have.

$$\underbrace{4 \text{ STATEMENTS}} \Rightarrow 2^4 = \boxed{16 \text{ ROWS}}$$

15. (6 points) Use truth tables to show that the statement  $p \rightarrow q$  is logically equivalent to  $\sim p \vee q$ .

P	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

P	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

IDENTICAL TRUTH TABLES



$p \rightarrow q$  IS EQUIVALENT TO  $\sim p \vee q$

16. (4 points) Use DeMorgan's Laws to write a logically equivalent statement.

(a)  $\sim(p \vee q)$

$$\sim p \wedge \sim q$$

(b)  $\sim(q \wedge r)$

$$\sim q \vee \sim r$$

17. (4 points) Write the negation of the statement "She drinks soda or tea."

SHE DOES NOT DRINK SODA, AND SHE DOES  
NOT DRINK TEA.

18. (12 points) By using truth tables, determine whether each statement is a tautology, a self-contradiction, or neither.

(a)  $(p \wedge \sim q) \wedge \sim p$

$p$	$q$	$\sim p$	$\sim q$	$p \wedge \sim q$	$(p \wedge \sim q) \wedge \sim p$
T	T	F	F	F	F
T	F	F	T	T	F
F	T	T	F	F	F
F	F	T	T	F	F

SELF-CONTRADICTION

(b)  $(p \rightarrow q) \vee \sim q$

$p$	$q$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \vee \sim q$
T	T	F	T	T
T	F	T	F	T
F	T	F	T	T
F	F	T	T	T

TAUTOLOGY

(c)  $(p \wedge q) \rightarrow p$

$p$	$q$	$p \wedge q$	$(p \wedge q) \rightarrow p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

TAUTOLOGY