Math 112 - Test 3
April 19, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (5 points) Jillian deposited \$700 into an investment account earning 6.75% simple interest. At the end of the investment period, she had \$841.75. How long was the investment period?

$I = 841.75 - 700 = 141.75$

$141.75 = 700(0.0675)t$

$\Rightarrow$ $t = 3$ YEARS

2. (5 points) Martin needs \$1829 to purchase furniture. The furniture store lends Martin the money at 11% simple interest for two years.

   (a) How much interest will Martin pay?

$I = 1829(0.11)(2) =$ \$402.38

   (b) Martin decides to pay the total amount (principal + interest) in 24 equal monthly payments. How much is each payment?

$1829 + 402.38 = 2231.38$

$2231.38 \div 24 \approx$ \$92.97

3. (5 points) What simple interest rate would be required to triple the value of your investment in 5 years?

TO TRIPLE AN INVESTMENT OF P DOLLARS YOU WOULD NEED $2P$ DOLLARS IN INTEREST.

$2P = P(r)(5)$

$\Rightarrow$ $r = \frac{2}{5} =$ 40%

4. (6 points) Determine the better investment: 3% compounded daily or 3.1% compounded quarterly. (Compute and compare the effective interest rates.)

3% DAILY: $E = \left(1 + \frac{0.03}{365}\right)^{365} - 1 \approx 3.045\%$

3.1% QUARTERLY: $E = \left(1 + \frac{0.031}{4}\right)^{4} - 1 \approx 3.14\%$

THIS ONE IS A BETTER RATE.

5. (8 points) A 25-year-old plans to retire at age 50. She decides to invest an inheritance of $80,000 at 7% interest compounded semiannually.

   (a) How much money will be in the account when she is 50 years old?

$$A = 80000\left(1 + \frac{0.07}{2}\right)^{2(25)}$$

$$= \$446,794.15$$

   (b) How much money is made in interest?

$$446,794.15 - 80,000$$

$$= \$366,794.15$$

6. (6 points) After winning $73,000 on a game show, Jasmine invests the money in a fixed-rate account offering 7.2% interest compounded quarterly. Use guess and check to determine about how long it will take for the account value to grow to $100,000.

$$100000 = 73000\left(1 + \frac{0.072}{4}\right)^{4t}$$

BY CHECKING DIFFERENT VALUES OF t, IT TURNS OUT THAT t IS BETWEEN 4.4 YEARS & 4.5 YEARS.

7. (6 points) A company needs to have \$4,000,000 in 10 years. The company will make semiannual payments into an account earning 8.75% compounded semiannually. How much will the semiannual payments be on the annuity?

$$R = \frac{4000000\left(\frac{0.0875}{2}\right)}{\left(\left(1+\frac{0.0875}{2}\right)^{2(10)}-1\right)} \approx \$129,182.90$$

8. (8 points) Suppose you open an annuity with quarterly payments of \$600 at 5% compounded quarterly for 15 years.

(a) Find the future value of the annuity.

$$A = \frac{600\left(\left(1+\frac{0.05}{4}\right)^{4(15)}-1\right)}{\left(\frac{0.05}{4}\right)} \approx \$53,144.70$$

(b) How much interest will you earn?

$$53144.70 - 60(600) = \$17,144.70$$

9. (6 points) Jamal has learned that he can get a new car by agreeing to make monthly payments of \$368 for five years. After reading the fine print, he realized that these monthly payments include a finance charge of 9.99% compounded monthly. How much would the car cost Jamal if he paid all at once in cash?

$$P = \frac{368\left(1-\left(1+\frac{0.0999}{12}\right)^{-12(5)}\right)}{\left(\frac{0.0999}{12}\right)} \approx \$17,324.07$$

10. (18 points) A house sells for \$186,450 and a 9% down payment is made. For the remaining balance, a 15-year mortgage is secured at 3.2% compounded monthly.

(a) What amount is financed?

$$\text{Down payment} = (0.09)(186450) = 16780.50$$

$$\text{Loan amount} = 186450 - 16780.50$$

$$= \boxed{\$169,669.50}$$

(b) What is the monthly payment?

$$R = \frac{169669.50\left(\frac{0.032}{12}\right)}{\left(1-\left(1+\frac{0.032}{12}\right)^{-180}\right)} \approx \boxed{\$1188.10}$$

(c) When the loan is paid off in 15 years, what will be the total interest paid?

$$180(1188.10) - 169669.50$$

$$= \boxed{\$44,188.50}$$

(d) Compute the first 3 rows of the amortization schedule. Include the payment number, interest, amount paid to principal, and the outstanding balance.

Loan amount \$169,669.50 Payment \$1188.10

Payment	Interest	Amount to Principal	Current Balance
1	452.45	735.65	168,933.85
2	450.49	737.61	168,196.24
3	448.52	739.58	167,456.66

11. (8 points) Compute each of the following.

(a) $7!$

$= 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = \boxed{5040}$

(b) $\dfrac{200!}{2!\,199!}$ $= \dfrac{200}{2} = \boxed{100}$

(c) ${}_6P_4$ $= \boxed{360}$

(d) ${}_6C_4$ $= \boxed{15}$

12. (3 points) A pizzeria offers single-topping pizzas with three choices of crust, two choices of sauce, and eight choices of toppings. How many different pizzas can be made?

$3 \times 2 \times 8 = \boxed{48}$

13. (6 points) Decide whether the selection described is a combination or a permutation.

(a) Five people in a meeting are selected to form a committee.

COMBINATION

(b) A state elects a governor, lieutenant governor, and treasurer from a pool of 10 candidates.

PERMUTATION

(c) A state elects two senators from a pool of 12 candidates.

COMBINATION

14. (4 points) How many different passwords can be made from the letters of the word *MISSISSIPPI*?

$$\frac{11!}{4!\,4!\,2!} = 34650$$

15. (6 points) In 5-card poker, each player is dealt 5 cards from a standard deck of 52 cards.

(a) How many different 5-card hands can be dealt?

$${}_{52}C_5 = 2,598,960$$

(b) How many different 5-card hands contain the ace of hearts?

$${}_{51}C_4 = 249,900$$

1. **Simple Interest Formulas**
   - $I = Prt$
   - $A = P + Prt$

2. **Compound Interest Formula**
   - $A = P\left(1 + \frac{r}{n}\right)^{nt}$

3. **Effective Rate**
   - $E = \left(1 + \frac{r}{n}\right)^{n} - 1$

4. **Annuity Formulas (Future value of payments)**
   - $A = \dfrac{R \cdot \left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}{\left(\frac{r}{n}\right)}$
   - $R = \dfrac{A \cdot \left(\frac{r}{n}\right)}{\left[\left(1 + \frac{r}{n}\right)^{nt} - 1\right]}$

5. **Present value of future payments**
   - $P = \dfrac{R \cdot \left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}{\left(\frac{r}{n}\right)}$

6. **Mortgage Formula (Payments for present value)**
   - $R = \dfrac{P \cdot \left(\frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{-nt}\right]}$

7. **Counting Formulas**
   - Permuations of n objects: $n!$
   - Permutations of r objects taken from n: ${}_nP_r = \dfrac{n!}{(n-r)!}$
   - Permutations of n objects where some are alike: $\dfrac{n!}{n_1!n_2! \cdots n_p!}$
   - Combinations of r objects taken from n: ${}_nC_r = \dfrac{n!}{(n-r)!\,r!}$