Math 112 - Final Exam

May 15, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (3 points) Let W be the set of all big words in Webster's dictionary. Is the set W well defined? Explain why or why not.

   No, since it is not clear what a "big" word is, we have no way of knowing what is or is not in W.

2. (6 points) The set S is defined below using set-builder notation.

   $$S = \{x |\ x \in \mathbb{N} \text{ and } 2x + 4 = 8\}$$

   $2x + 4 = 8 \Rightarrow x = 2$

   (a) Write S in roster notation.

   $S = \{2\}$

   (b) What is the cardinality of S?

   $n(S) = 1$

   (c) Give an example of a set that is equivalent to S, but not equal to S.

   $\{a\}$

3. (2 points) Let R be the set of even natural numbers between 13 and 23. Write R in roster notation.

   $R = \{14, 16, 18, 20, 22\}$

4. (3 points) List all proper subsets of the set $\{a, b\}$.

   $\emptyset, \{a\}, \{b\}$

5. (10 points) Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 4, 8\}$, and $B = \{1, 2, 3, 4\}$.

(a) Is it true that $B \subseteq A$? If not, explain why.

↳ No, because $3 \in B$ but $3 \notin A$.

(b) Determine $A \cup B$.

$\{1, 2, 3, 4, 8\}$

(c) Determine $A - B$.

$\{8\}$

(d) Which of the following is **not** a subset of B? Circle a single choice.

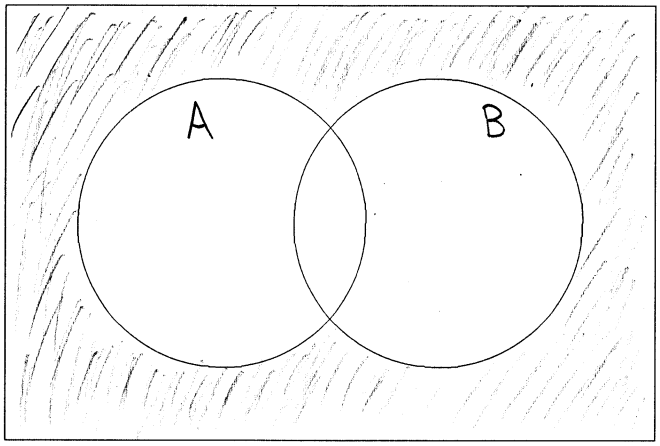
$\{1, 2, 3, 4\}$ $\quad$ $\{\emptyset\}$ $\quad$ $\{1\}$ $\quad$ $\emptyset$

(e) Determine $n(B')$. $B' = \{0, 5, 6, 7, 8, 9\}$

$n(B') = 6$

6. (5 points) Label the circles and shade the region corresponding to $A' \cap B'$.

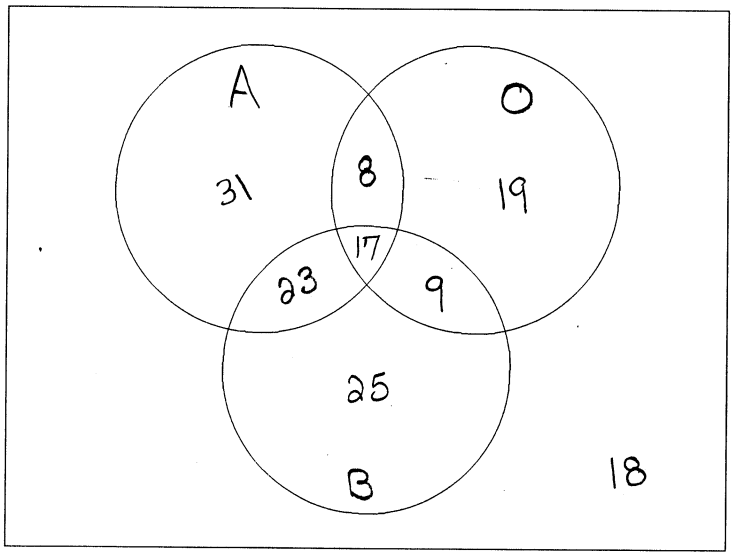
$= (A \cup B)'$



7. (16 points) Mr. Strand's 4th grade class conducted a survey in which 150 students were asked about the kinds of fruit they normally eat. Here is what they found:

- 79 eat apples
- 53 eat oranges
- 74 eat bananas
- 25 eat apples and oranges
- 40 eat apples and bananas
- 26 eat oranges and bananas
- 17 eat all three

(a) Count and sort these results using a three-set Venn diagram.



(b) How many students do not eat any of these fruits?

18

(c) How many students eat two of the fruits, but not all three?

$23 + 8 + 9 = 40$

(d) How many students eat only one kind of fruit?

$31 + 19 + 25 = 75$

8. (8 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

(a) I'm going to pass my math class if and only if I do my homework.

BICONDITIONAL

(b) All dogs are mammals.

CONDITIONAL

(c) When your battery dies, you should charge your phone.

CONDITIONAL

(d) Sara and Brian went hiking in the mountains.

CONJUNCTION

9. (6 points) Let p = "The plane is on time." and let q = "The sky is clear." Write each statement in words.

(a) $q \longrightarrow p$

IF THE SKY IS CLEAR, THEN THE PLANE IS ON TIME.

(b) $\sim p \wedge q$

THE PLANE IS NOT ON TIME AND THE SKY IS CLEAR.

10. (8 points) Construct the truth table for $(\sim q \vee p) \longrightarrow p$.

p	q	$\sim q$	$\sim q \vee p$	$(\sim q \vee p) \longrightarrow p$
T	T	F	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	T	F

11. (10 points) Consider the following conditional statement:

If today is Wednesday, then pizza is on the menu.

(a) State the inverse.

IF TODAY IS NOT WEDNESDAY, THEN PIZZA IS NOT ON THE MENU.

(b) State the contrapositive.

IF PIZZA IS NOT ON THE MENU, THEN TODAY IS NOT WEDNESDAY.

(c) State the converse.

IF PIZZA IS ON THE MENU, THEN TODAY IS WEDNESDAY.

(d) Of the three, which is logically equivalent to the original statement?

Inverse · (Contrapositive) · Converse

12. (8 points) Sylvia needs \$1829 to purchase furniture. The furniture store lends Sylvia the money at 11% simple interest for three years.

(a) How much interest will Sylvia pay?

$1829(0.11)(3) = \$603.57$

(b) Sylvia decides to pay the total amount (principal + interest) in 36 equal monthly payments. How much is each payment?

$$\frac{1829 + 603.57}{36} = \$67.57$$

13. (8 points) Suppose you begin depositing monthly payments into an account earning 4.5% compounded quarterly. Your goal is to accumulate \$20,000 in 8 years. What should your monthly payments be?

QUARTERLY

QUARTERLY

$$\frac{20000\left(\frac{0.045}{4}\right)}{\left(\left(1+\frac{0.045}{4}\right)^{(4\cdot 8)}-1\right)} = \$522.71$$

14. (10 points) Brenda has been offered a loan at 5.85% compounded monthly which requires monthly payments of \$465.75 for 5 years. What is the loan amount? When the loan is paid off, how much interest will Brenda have paid?

$$\frac{465.75\left(1-\left(1+\frac{0.0585}{12}\right)^{-(12\cdot 5)}\right)}{\left(\frac{0.0585}{12}\right)} = \$24,178.31$$

$$\text{INTEREST} = 60(465.75) - 24178.31 = \$3766.69$$

15. (12 points) A house sells for \$182,600 and an 6% down payment is made. A mortgage is secured for the remaining amount for 30 years at 3.75% compounded monthly.

(a) What amount is financed?

$$\text{DOWN PAYMENT} = 0.06(182600) = 10,956$$

$$182,600 - 10,956 = \$171,644.00$$

(b) What is the monthly payment?

$$\frac{171644\left(\frac{0.0375}{12}\right)}{\left(1-\left(1+\frac{0.0375}{12}\right)^{-360}\right)} = \$794.91$$

(c) When the loan is paid off in 30 years, what will be the total interest paid?

$$360(794.91) - 171644 = \$114,523.60$$

16. (4 points) Stephanie is looking for a new case for her phone. She looks at 7 different types of case, and each comes in 5 different colors. How many different choices does Stephanie have?

$7 \times 5 = 35$

17. (8 points) Joe's pocket contains 12 pennies, 8 nickels, and 5 dimes. He picks a coin at random.

$12 + 8 + 5 = 25$

(a) What is the probability of picking a penny?

$\frac{12}{25} = 0.48$

(b) What is the probability of picking a nickel or a dime?

$\frac{13}{25} = 0.52$

(c) What are the odds in favor of picking a nickel?

$\frac{8}{25-8} = \frac{8}{17}$

(d) What are the odds against picking a nickel?

$\frac{17}{8}$

18. (5 points) A woman draws a letter at random from the word *MISSISSIPPI*. If she draws the letter *S*, she wins \$3. If she draws *I*, she wins \$3. If she draws *P*, she wins \$10. And if she draws *M*, she wins \$25. What is the expected value for this game?

$P(\{M\}) = \frac{1}{11}$

$P(\{I\}) = \frac{4}{11}$

$P(\{S\}) = \frac{4}{11}$

$P(\{P\}) = \frac{2}{11}$

$3\left(\frac{4}{11}\right) + 3\left(\frac{4}{11}\right) + 10\left(\frac{2}{11}\right) + 25\left(\frac{1}{11}\right)$

$= \frac{12+12+20+25}{11} = \frac{69}{11}$

$\approx \$6.27$

19. (4 points) Kevin's personal experience suggests that the odds in favor of the copy machine jamming are 2 to 7. What are the odds against a jam? What is the probability of a jam?

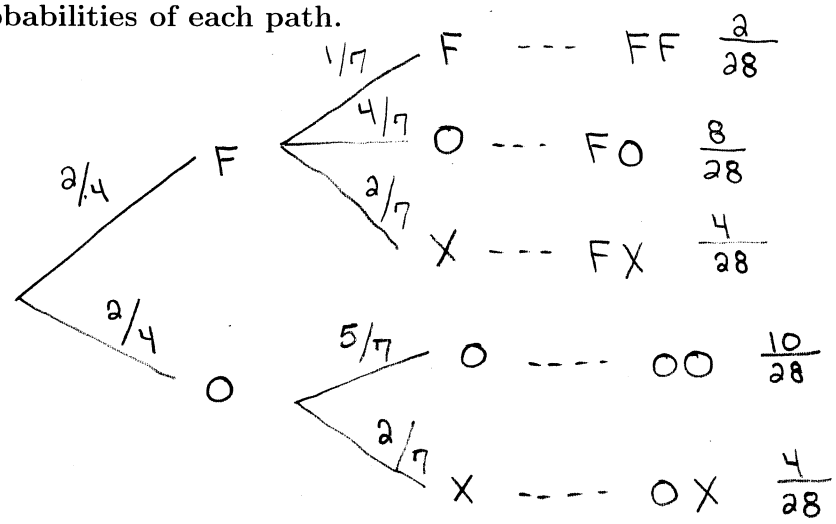
$\frac{7}{2}$ $\qquad$ $\frac{2}{9}$

20. (14 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.

F	F	O	O

O	O	O	O	X	X

(a) Sketch the complete tree diagram for this experiment. **Include the probabilities of each path.**



(b) What is the probability of selecting at least one X?

$\frac{4}{28} + \frac{4}{28} = \frac{8}{28}$

(c) What is the probability of selecting F from the first box or O from the second box?

$\{FF, FO, FX, OO\}$

$1 - \frac{4}{28} = \frac{24}{28}$