Math 112 - Test 2

March 21, 2019

Name __key__

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Which of these sentences are statements? Circle all that apply.

   (a) Please leave the room.

   (b) Maryam Mirzakhani is the only woman to have won the Fields Medal in mathematics.

   (c) Welcome back!

   (d) 19 is a multiple of 3.

   (e) Mathematician David Blackwell was the first African-American faculty member at UC Berkeley.

   (f) Were you late for class?

2. (5 points) Identify each as a conjunction, disjunction, conditional, or biconditional.

   (a) She finished her thesis and received her degree.

   ↑ CONJUNCTION

   (b) He will be the first Hispanic president if he gets elected.

   ↑ CONDITIONAL

   (c) I watched *Hidden Figures*, but I had already read the book.

   ↑ AND   CONJUNCTION

   (d) Katherine will catch a cold if and only if she leaves the window open.

   ↑ BICONDITIONAL

   (e) I'll ride my bike or go for a run.

   ↑ DISJUNCTION

3. (2 points) Write an example of a quantified statement.

   SOME STUDENTS ARE MATH MAJORS.

   ↑ QUANTIFIER

1

4. (8 points) Write the negation of each statement in a correct sentence.

   (a) Some rectangles are not squares.

   ALL RECTANGLES ARE SQUARES.

   (b) Nobody will ever run a 3-minute mile.

   SOME ONE WILL RUN A 3-MIN MILE.

   (c) His last name has only two letters.

   HIS LAST NAME DOES NOT HAVE ONLY TWO LETTERS.

   (d) Someone in this class will get an A.

   NO ONE IN CLASS WILL GET AN A.

5. (6 points) Let p = "Sam will get a job." and let q = "Sam will buy a new car." Write each statement in symbolic form.

   (a) Sam will get a job, but he won't buy a new car.

   $p \land \sim q$

   (b) Sam will buy a new car if he will get a job.

   $p \rightarrow q$

   (c) If Sam won't buy a new car, then he won't get a job.

   $\sim q \rightarrow \sim p$

6. (6 points) Refer to the statements p and q from the problem directly above. Write each statement in words.

   (a) $\sim p \land \sim q$ SAM WON'T GET A JOB AND HE WON'T BUY A NEW CAR.

   (b) $p \longleftrightarrow p$

   SAM WILL GET A JOB IF AND ONLY IF HE WILL GET A JOB.

   (c) $\sim q \lor p$

   SAM WON'T BUY A NEW CAR OR HE WILL GET A JOB.

2

7. (4 points) Determine the truth value of the following statement: *The current year is before 1970 if and only if there are fewer than a million people in the United States.* Show work or explain.

$$F \longleftrightarrow F \quad \text{is} \quad \boxed{\text{TRUE}}$$

8. (4 points) Write the negation of the statement "Linda studies math, and Jon does not study physics." (Hint: Use DeMorgan's laws.)

LINDA DOES NOT STUDY MATH, OR

JON STUDIES PHYSICS.

9. (2 points) If q is true, what is the truth value of $\sim(\sim(\sim q))$?

not (not (not T))

$$= \text{not (not F)} = \text{not (T)} = \boxed{F}$$

10. (4 points) Without actually constructing it, determine how many rows and columns the truth table for $(p \vee q) \wedge (p \to r)$ would have. (Label which answer is which.)
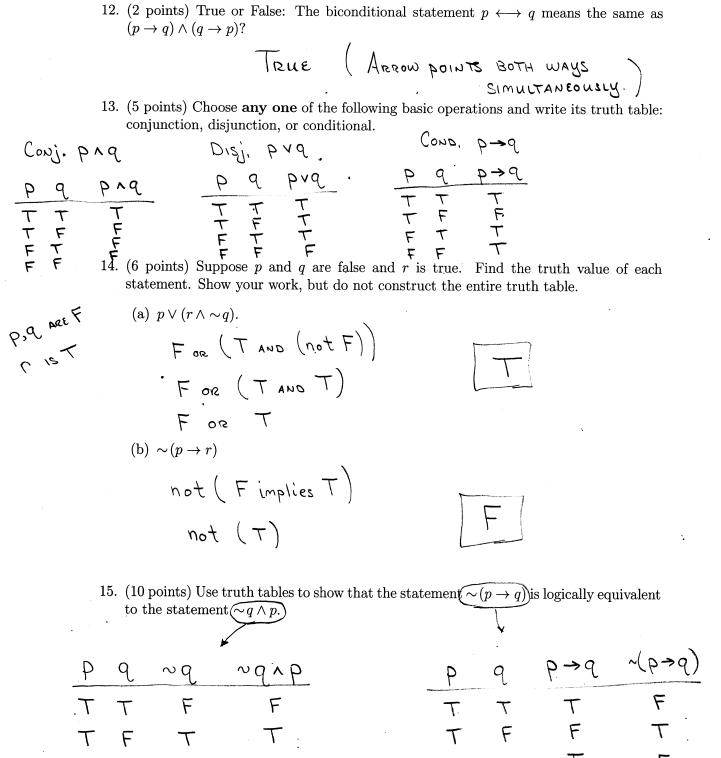
3 STATEMENTS

3 OPS $\Rightarrow$ Rows: $2^3 = 8$

Columns: $3 + 3 = 6$

11. (4 points) Write a logically equivalent statement without parentheses. (Hint: Use DeMorgan's laws.)

(a) $\sim(p \vee q)$

$$\sim p \wedge \sim q$$

(b) $\sim(q \wedge \sim r)$

$$\sim q \vee r$$

12. (2 points) True or False: The biconditional statement $p \longleftrightarrow q$ means the same as $(p \to q) \land (q \to p)$?

TRUE (ARROW POINTS BOTH WAYS SIMULTANEOUSLY.)

13. (5 points) Choose **any one** of the following basic operations and write its truth table: conjunction, disjunction, or conditional.

Conj. $p \land q$

p	q	$p \land q$
T	T	T
T	F	F
F	T	F
F	F	F

Disj. $p \lor q$

p	q	$p \lor q$
T	T	T
T	F	T
F	T	T
F	F	F

Cond. $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

14. (6 points) Suppose p and q are false and r is true. Find the truth value of each statement. Show your work, but do not construct the entire truth table.

p,q ARE F
r IS T

(a) $p \lor (r \land \sim q)$.

F OR (T AND (not F))

F OR (T AND T)

F OR T

$\boxed{T}$

(b) $\sim(p \to r)$

not (F implies T)

not (T)

$\boxed{F}$

15. (10 points) Use truth tables to show that the statement $\sim(p \to q)$ is logically equivalent to the statement $\sim q \land p$.

p	q	$\sim q$	$\sim q \land p$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

p	q	$p \to q$	$\sim(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

IDENTICAL TRUTH TABLES.

$\sim(p \to q) \equiv \sim q \land p$

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16. (8 points) Consider the following conditional statement:

$$\overbrace{\text{If she graduates,}}^{P} \text{ then } \overbrace{\text{she starts work at NASA.}}^{q}$$

(a) State the converse. $q \to p$

IF SHE STARTS WORK AT NASA, THEN SHE GRADUATES.

(b) State the inverse. $\sim p \to \sim q$

IF SHE DOES NOT GRADUATE, THEN SHE DOES NOT START WORK AT NASA.

(c) State the contrapositive. $\sim q \to \sim p$

IF SHE DOES NOT START WORK AT NASA, THEN SHE DOES NOT GRADUATE.

(d) Which statement is logically equivalent to the original statement?

Inverse   (Contrapositive)   Converse

(e) Which statement is logically equivalent to the inverse?

(Inverse)   Contrapositive   Original

(CONVERSE)

17. (3 points) There are several mistakes in the truth table shown below. Correct the mistakes.

p	q	$\sim q$	$(\sim q \lor p)$	$(\sim q \lor p) \to q$
T	T	F	(F) T	T
T	F	T	T	F
F	T	(T) F	F	T
F	F	T	T	(T) F

18. (15 points) By using truth tables, determine whether each statement is a tautology, a self-contradiction, or neither.

(a) $(p \vee q) \to q$

p	q	p ∨ q	(p ∨ q) → q
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	T

NEITHER

(b) $(p \wedge q) \wedge \sim q$

p	q	p ∧ q	∼q	(p ∧ q) ∧ ∼q
T	T	T	F	F
T	F	F	T	F
F	T	F	F	F
F	F	F	T	F

SELF CONTRADICTION

(c) $p \to (p \wedge q)$

p	q	p ∧ q	p → (p ∧ q)
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

NEITHER