Math 130 - Final Exam

December 11, 2019

Name _____

Score _____

Show all work to receive full credit. Supply explanations where necessary. When finding exact answers, simplify as much as possible. You may use your unit circle and trig identity card on any problem. For each triangle described below, a is opposite α, b is opposite β, and c is opposite γ (unless otherwise indicated).

---

1. (3 points) Convert $285°$ to radian measure. Write your answer as a fraction of π in lowest terms.

2. (10 points) Sketch a right triangle with an acute angle θ for which $\cot \theta = \frac{3}{2}$. Then find the values of the other five trigonometric functions at θ. You do not have to rationalize your denominators, but otherwise write your fractions as simple as possible.

3. (6 points) $\tan \beta = 12/5$ and $\sin \beta < 0$. Find the exact values of $\sin \beta$ and $\cos \beta$.

4. (8 points) On the attached graph paper, sketch the graph of $y = 3\cos(2x + \pi)$. Label your graph well enough for a person to read it. Include two full periods.

5. (10 points) Use your knowledge of the values of the trigonometric functions at special angles to determine the exact value of each of the following. Do not use a calculator. Show or explain how you got each answer.

   (a) $\cos^{-1}\left(-\dfrac{1}{2}\right)$
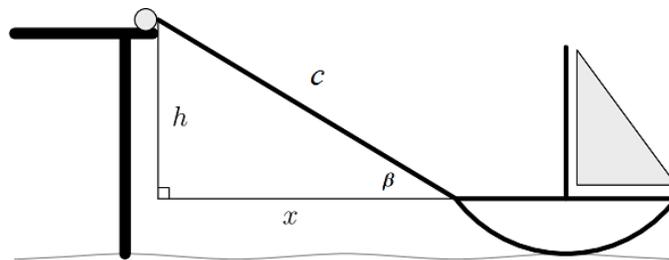
   (b) $\tan^{-1} 1$

   (c) $\arcsin\left(\dfrac{-1}{2}\right)$

   (d) $\sin^{-1} 1$

   (e) $\arccos 2$

6. (4 points) Determine the exact value of $\sin^{-1}[\sin(23\pi/6)]$.

7. (6 points) Use a right triangle to find the exact value of $\sin(\tan^{-1}(5/9))$.

8. (5 points) A boat is being pulled toward a dock as shown in the figure below. Determine the angle β if $x = 6\,\text{m}$ and $h = 2\,\text{m}$. Give your answer in degree measure, rounded to the nearest hundredth of a degree.

9. (6 points) Verify the identity: $\dfrac{\sec^2\theta - 1}{\sec^2\theta} = \sin^2\theta$

10. (8 points) Find the exact solutions in the interval $[0, 2\pi)$: $\quad 2\sin^2 x - 3\sin x + 1 = 0$

11. (10 points) Given that β is a 2nd quadrant angle with $\cos\beta = -4/5$, find the exact values of $\sin 2\beta$, $\cos 2\beta$, and $\tan 2\beta$. Do not use a calculator for this problem.

12. (6 points) Solve the triangle: $\quad \alpha = 85°, \quad a = 15, \quad b = 25$

13. (10 points) Solve the triangle: $\alpha = 25°, \quad b = 9, \quad c = 12$

14. (6 points) Let $z_1 = 3(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})$ and $z_2 = 4(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})$.

    (a) Find the product $z_1 z_2$ in polar form.

    (b) Write the product $z_1 z_2$ in standard form.

15. (6 points) Let $z = 1 + i$. Write z in polar form. Then use DeMoivre's theorem to compute z^6. Write your final answer in standard form.

16. (9 points) Determine five points on the graph of $f(x) = 2^x - 3$. Then plot your points and carefully sketch the graph. Indicate any asymptotes.
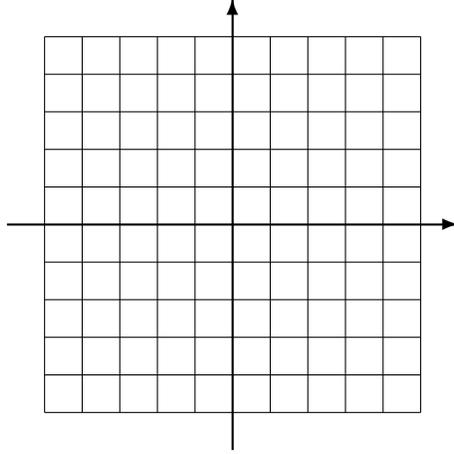
17. (2 points) Rewrite as a logarithmic equation: $4^5 = 1024$

18. (2 points) Rewrite as an exponential equation: $\log_7 343 = 3$

19. (4 points) Use properties of logarithms to completely expand: $\ln\left(\dfrac{x^4 y}{\sqrt{z^5}}\right)$

20. (3 points) Use the change-of-base formula to write $\log_7 50$ in terms of natural logarithms. Then use your calculator to compute the value. Round to the nearest thousandth.

21. (8 points) Solve for x: $\quad \log 5x + \log(x-1) = 2$

22. (8 points) Solve for x: $\quad 4^x = 5^{x^2}$

23. (10 points) Polonium-210 has a half-life of 140 days. Use a model of the form $P(t) = Ae^{-kt}$ to determine the initial amount of Polonium-210 if there were 2.5 grams remaining after 600 days.