Math 132 - Test 3

April 16, 2020

Name _____

Score _____

Show all work to receive full credit. Supply explanations when necessary. You must work individually on this test. Submit your test in Blackboard no later than Monday, April 20, at 9pm.

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1. (5 points) In a single complete sentence, clearly explain why the integral is improper. Then write it as the limit of a proper integral. Do not evaluate.

$$\int_8^{12} \frac{x-12}{\sqrt{x-8}}\,dx$$

2. (5 points) Use the trapezoid rule with four subintervals ($n = 4$) to approximate the value of the following integral. When you round, use at least 6 decimal places.

$$\int_1^2 \sin\left(\frac{1}{x}\right)\,dx$$

3. (5 points) Write as the limit of a proper integral and evaluate: $\displaystyle\int_0^\infty \frac{4x}{x^2+1}\,dx$

4. (5 points) Consider the sequence $\{a_n\}_{n=1}^{\infty}$, where $a_1 = 2$, $a_2 = 4$, and for $n > 2$, $a_n = \frac{1}{2}(a_{n-1} + a_{n-2})$. Write the first six terms of the sequence. Based on your terms, do you think the sequence converges or diverges?

5. (5 points) Find the limit of the sequence whose nth term is $a_n = \dfrac{(n-1)^2}{(2n+1)^2}$.

6. (5 points) What does it mean for a series to converge?

7. (5 points) The series $\sum_{n=0}^{\infty} \frac{2}{5^n}$ is a special kind of series. What is its name? State whether it converges or diverges (and how you know). If it converges, find its sum.

8. (5 points) The series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2} \right)$ is a telescoping series. Find a formula for the nth partial sum.

9. (10 points) Consider the series $\sum_{n=1}^{\infty} \frac{n}{2n^2+3}$.

(a) Apply the nth term test for divergence and describe the conclusion of the test.

(b) Apply the limit comparison test by comparing the series with the harmonic series, $\sum_{n=1}^{\infty} \frac{1}{n}$. What is your conclusion?

10. (5 points) Give an example of a convergent p-series and a divergent p-series. Say how you know and which is which.

11. (5 points) Steve was asked to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{3^n - n}$. He used the direct comparison test to compare his series with the convergent geometric series $\sum_{n=1}^{\infty} \frac{1}{3^n}$. What is wrong with Steve's approach?

12. (5 points) The integral test cannot be used to test the series $\sum_{n=0}^{\infty} e^{-n} \sin n$. Why not?

13. (5 points) Use the alternating series test to determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges or diverges.

14. (5 points) Does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{5/4}}$ converges conditionally or absolutely? How do you know?

15. (10 points) It was first discovered by Indian mathematicians in the 1300's that

$$\frac{\pi}{4} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}.$$

(a) Use your calculator (or computer) to compute S_{20}, the 20th partial sum of the series. Round your final answer to 6 decimal places.

(b) Use the alternating series remainder theorem to determine an upper bound on the error made in using the approximation $\frac{\pi}{4} \approx S_{20}$.

16. (5 points) Think about the series $\sum_{n=1}^{\infty} \frac{1}{\pi^n}$. Tamara says that the series converges, and Jon says that the series converges absolutely. Who is correct? And why?

17. (5 points) Looking back that series above, $\sum_{n=1}^{\infty} \frac{1}{\pi^n}$, you should recognize that it is a geometric series. Explain what is wrong with using the formula

$$\sum_{n=1}^{\infty} \frac{1}{\pi^n} = \frac{1}{1-\pi}.$$

18. (5 points) As briefly as possible, tell the difference between a sequence and a series.