Math 151 - Test 1
September 16, 2015

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Write the relation $\frac{x+3y}{2} = 5$ as a function of x. Then state the domain and range.

$x + 3y = 10$

$3y = -x + 10$

$y = \frac{-x+10}{3}$

THIS IS A NONCONSTANT LINEAR FUNCTION.

DOMAIN: $(-\infty, \infty)$

RANGE: $(-\infty, \infty)$

2. (2 points) What is the domain of the function $f(x) = \frac{x}{x+7}$?

DIVISION BY ZERO IS NOT DEFINED.

$x \neq -7$

DOMAIN IS $(-\infty, -7) \cup (-7, \infty)$

3. (6 points) Let $g(x) = x^2 - 4x$. Compute and simplify the difference quotient $\frac{g(a+h) - g(a)}{h}$.

$g(a+h) = (a+h)^2 - 4(a+h)$

$= a^2 + 2ah + h^2 - 4a - 4h$

$$\frac{g(a+h) - g(a)}{h} = \frac{[a^2 + 2ah + h^2 - 4a - 4h] - [a^2 - 4a]}{h}$$

$$= \frac{2ah + h^2 - 4h}{h} = 2a + h - 4$$

4. (6 points) Determine a formula for the linear function whose graph passes through the points $(8, 3)$ and $(-2, -12)$.

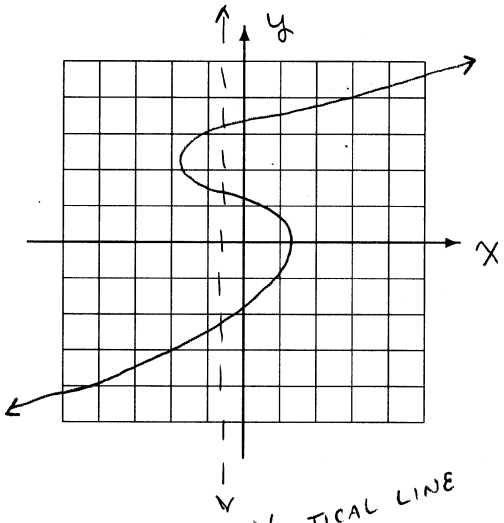
$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-12-3}{-2-8} = \frac{-15}{-10} = \frac{3}{2}$$

$$f(x) = \frac{3}{2}x + b$$

$$f(8) = \frac{3}{2}(8) + b = 12 + b = 3 \Rightarrow b = -9$$

$$\boxed{f(x) = \frac{3}{2}x - 9}$$

5. (3 points) Sketch the graph of a relation that is NOT a function. Explain why it is not a function.

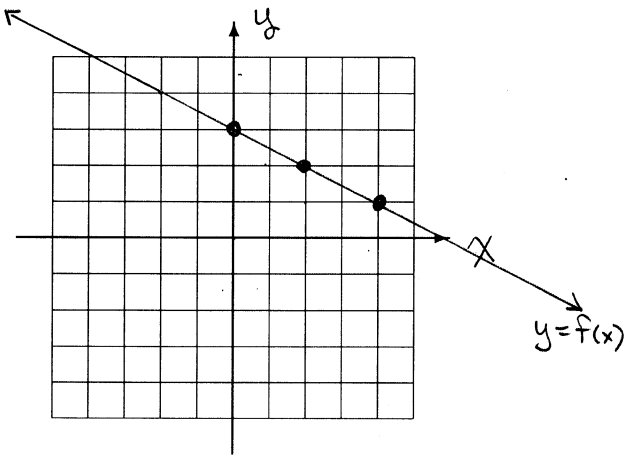


VERTICAL LINE CROSSES AT SEVERAL PLACES

NOT A FUNCTION BECAUSE SOME X-VALUES ARE PAIRED WITH MULTIPLY Y-VALUES.

GRAPH FAILS THE VERTICAL LINE TEST.

6. (6 points) Find at least three points on the graph of $f(x) = -\frac{1}{2}x + 3$. Then sketch the graph. (Label your axes.)



x	f(x)
0	3
2	2
4	1

7. (5 points) Sitting in a tree, 26 feet above ground level, Sue shoots a pebble straight up with a velocity of 80 feet per second. The height of the pebble, in feet, after t seconds is given by

$$h(t) = -16t^2 + 80t + 26.$$

What is the maximum height attained by the pebble? Give units on your answer.

VERTEX AT $t = \frac{-80}{2(-16)} = \frac{80}{32} = 2.5$

$h(2.5) = -16(2.5)^2 + 80(2.5) + 26 = \boxed{126 \text{ FT}}$

8. (4 points) Find the x- and y-intercepts of the graph of $g(x) = x^2 - x - 6$.

$g(x) = 0 \Rightarrow x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, \; x = -2$

X-INTERCEPTS ARE $(3,0)$ & $(-2,0)$

$g(0) = -6$

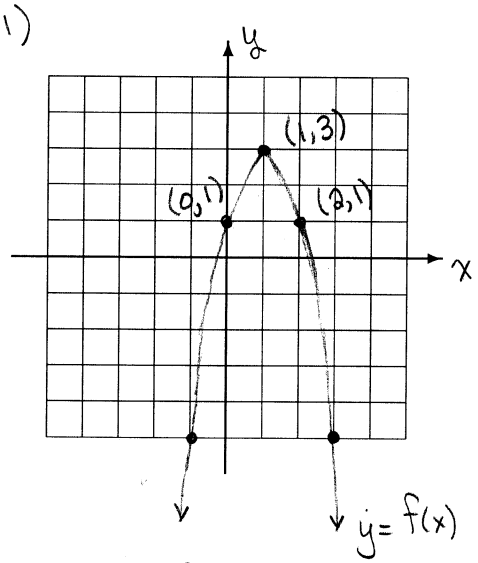
Y-INTERCEPT IS $(0,-6)$

9. (10 points) Let $f(x) = -2(x-1)^2 + 3$. Determine the graph's vertex and two other points on the graph. Then carefully sketch the graph and determine the range of f. (Label your axes.)

VERTEX AT $(1,3)$

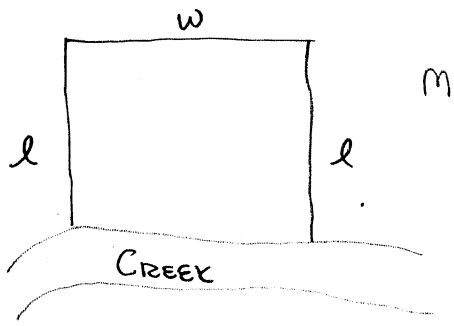
$x = 2 \Rightarrow y = f(2) = 1 \quad (2,1)$

$x = 0 \Rightarrow y = f(0) = 1 \quad (0,1)$



RANGE IS $(-\infty, 3]$

10. (10 points) The back of George's property is a creek. George would like to enclose a rectangular area, using the creek as one side and fencing for the other three sides, to create a pasture for his two horses. He has 300 feet of fencing material. Find the dimensions of the rectangle that maximize the enclosed area. (You must show all work for full credit.)



$w + 2\ell = 300 \Rightarrow w = 300 - 2\ell$

Maximize $A = \ell w$

$A = \ell(300 - 2\ell)$

$= -2\ell^2 + 300\ell$

Vertex at

$\ell = \frac{-300}{-2(2)} = 75$

$\ell = 75 \Rightarrow w = 150$

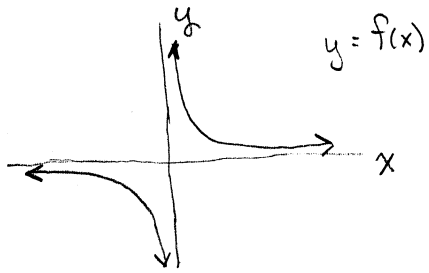
Dimensions are

$\ell = 75$ ft

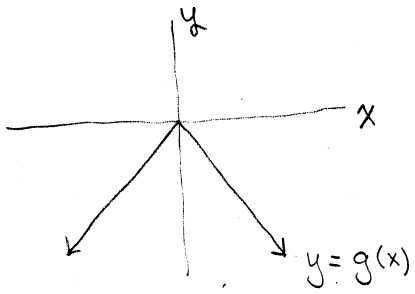
$w = 150$ ft

11. (6 points) Very roughly, sketch the general shape of the graph of each function.

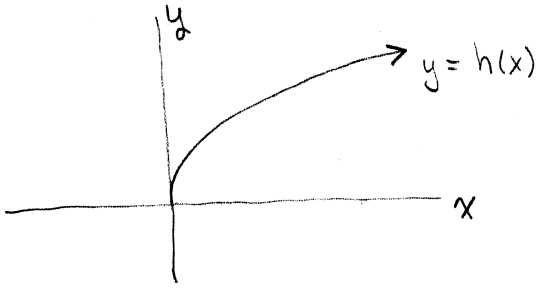
(a) $f(x) = \dfrac{2}{x^5}$



(b) $g(x) = -\dfrac{|x|}{3}$



(c) $h(x) = \sqrt[4]{x}$



12. (8 points) Consider the function

$$f(x) = \begin{cases} 4x^2 - 10, & x \leq 2 \\ x^3 - 5x + 1, & x \geq 3 \end{cases}$$

(a) Evaluate $f(4)$.

$f(4) = (4)^3 - 5(4) + 1 = 64 - 20 + 1 = \boxed{45}$

(b) Evaluate $f(0)$.

$f(0) = 4(0)^2 - 10 = \boxed{-10}$

(c) What is the value of $f(2.5)$?

NOT DEFINED

(d) What is the domain of f?

$(-\infty, 2] \cup [3, \infty)$

13. (6 points) Describe how the graph of $g(x) = -(x+3)^3 - 5$ can be obtained from the graph of $f(x) = x^3$.

① SHIFT LEFT 3 UNITS ② FLIP / REFLECT ABOUT X-AXIS ③ SHIFT DOWN 5 UNITS

14. (4 points) The graph of $h(x) = \sqrt{x}$ is shifted 8 units left and 4 units up to create the graph of a new function. What is that new function?

$f(x) = \sqrt{x+8} + 4$

15. (4 points) Some values of the functions f and g are given in the table below. Use the data from the table to evaluate each of the following.

x	1	2	3	4	5
$f(x)$	4	-2	8	0	1
$g(x)$	2	2	4	-7	-9

(a) $(f+g)(4)$

$= f(4) + g(4) = 0 + (-7) = \boxed{-7}$

(b) $(gf)(2)$

$g(2) \cdot f(2) = 2(-2) = \boxed{-4}$

(c) $(g-f)(5)$

$g(5) - f(5) = -9 - 1 = \boxed{-10}$

(d) $\left(\frac{g}{f}\right)(4) = \frac{g(4)}{f(4)}$ NOT DEFINED SINCE $f(4) = 0$

16. (6 points) Let $r(x) = \sqrt{x+6}$ and $s(x) = x^2 + |x| - 1$. Compute each of the following.

(a) $(r-s)(-2)$

$r(-2) - s(-2) = \sqrt{4} - (4 + 2 - 1) = 2 - 5 = \boxed{-3}$

(b) $(s+r)(3)$

$s(3) + r(3) = (9 + 3 - 1) + \sqrt{9} = 11 + 3 = \boxed{14}$

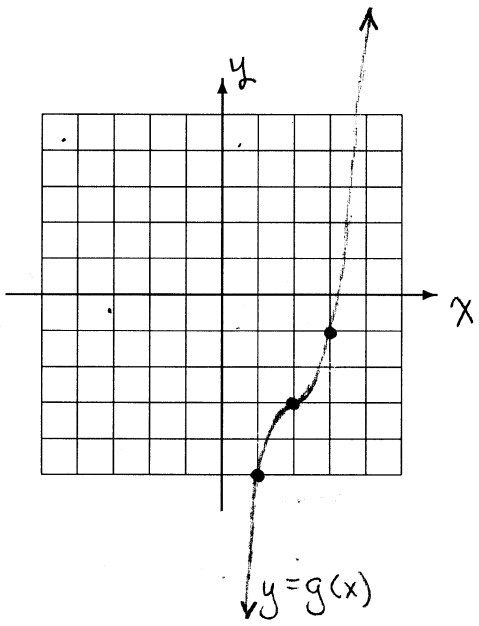
(c) $(sr)(-5)$

$s(-5) \cdot r(-5) = (25 + 5 - 1) \cdot \sqrt{1} = \boxed{29}$

17. (8 points) **Carefully sketch** the graph of each function. (Label your axes.)

(a) $g(x) = 2(x-2)^3 - 3$

x	$g(x)$
2	-3
1	-5
3	-1
0	-19



(b) $f(x) = 4 - \frac{3}{2}|x|$

x	$f(x)$
0	4
-2	1
2	1
4	-2
-4	-2