

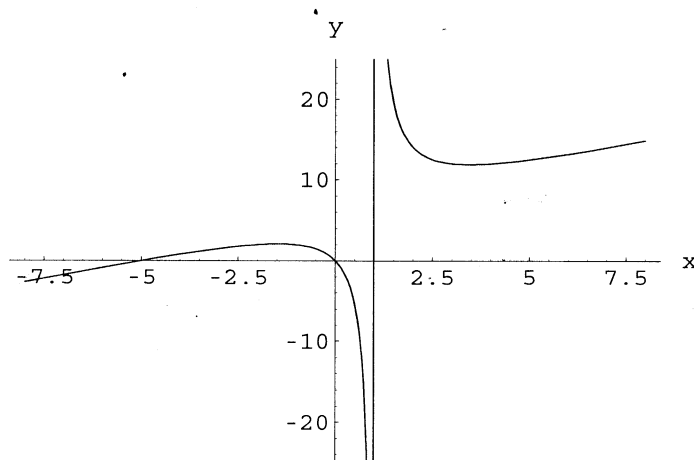
**Math 151 - Test 3**

November 18, 2015

Name key Score \_\_\_\_\_

Show all work. Supply explanations where necessary.

1. (5 points) The graphs of the function  $h$  and its vertical asymptote are shown below. Use the graph to solve the inequality  $h(x) \geq 0$ . Write your solution in interval notation.



$h(x) \geq 0$   
WHERE GRAPH LIES  
ABOVE OR ON  
X-AXIS.

$$[-5, 0] \cup (1, \infty)$$

2. (8 points) The half-life of radioactive carbon-14 is about 5700 years. Assume that carbon-14 decays exponentially so that the amount after  $t$  years satisfies  $A(t) = A_0 a^t$ , where  $A_0$  is the initial amount.

(a) Find  $a$ . Round your result to 6 decimal places.

$$\frac{1}{2} A_0 = A_0 a^{5700} \Rightarrow \frac{1}{2} = a^{5700} \Rightarrow a = \left(\frac{1}{2}\right)^{1/5700} \approx 0.999878$$

(b) A sample initially contains 85 grams of carbon-14. How much remains after 4000 years?

$$A(t) = 85 (0.999878)^t$$

$$A(4000) = 85 (0.999878)^{4000} \approx 52.2 \text{ grams}$$

3. (12 points) Consider the rational function  $R(x) = \frac{x+4}{x^2-4} = \frac{x+4}{(x+2)(x-2)}$

(a) What numbers are excluded from the domain of  $R$ ?

$$x = -2; x = 2$$

(b) Find the  $x$ - and  $y$ -intercepts of the graph of  $R$ .

$x$ -INT :

$$(-4, 0)$$

$y$ -INT :

$$(0, -1)$$

(c) Find the equation of any horizontal or slant asymptote of the graph of  $R$ .

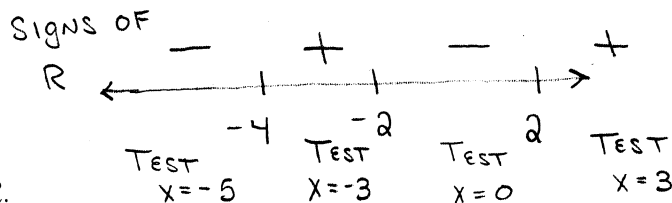
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$$\Rightarrow y = 0$$

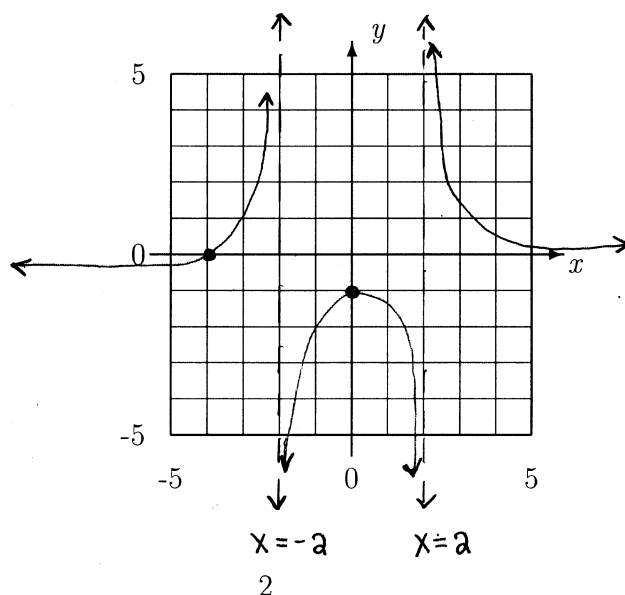
(d) Find the equations of any vertical asymptotes of the graph of  $R$ .

$$x = -2, x = 2$$

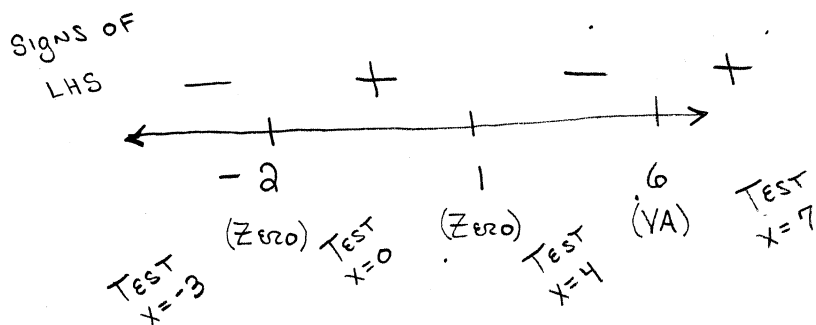
(e) Construct a sign chart showing the signs of  $R$  on the intervals bounded by the zeros of  $R$  and the points excluded from the domain of  $R$ .



(f) Sketch the graph of  $R$ .



4. (7 points) Solve the inequality  $\frac{(x-1)(x+2)}{x-6} \leq 0$ . Write your solution in interval notation.



LHS  $\leq 0$   
ON

$$(-\infty, -2] \cup [1, 6)$$

INCLUDE ZEROS / EXCLUDE VA

5. (4 points) Give an example of a rational function whose graph has vertical asymptotes  $x = 1$  and  $x = 2$  and  $x$ -intercepts  $(3, 0)$  and  $(-5, 0)$ .

$$R(x) = \frac{(x-3)(x+5)}{(x-1)(x-2)}$$

6. (4 points) Determine the slant asymptote of the graph of  $F(x) = \frac{x^2 - 9x + 4}{x + 2} = x - 11 + \frac{26}{x + 2}$

$$\begin{array}{r} -2 \overline{) 1 \ -9 \ 4} \\ \underline{-2 \ 22} \\ 1 \ -11 \ 26 \end{array}$$

SLANT ASYMP  
IS

$$y = x - 11$$

7. (8 points) \$2500 is deposited into a bank account that earns 5.5% per year.

- (a) Find the function of the form  $A(t) = A_0 a^t$  that describes the amount of money in the bank after  $t$  years.

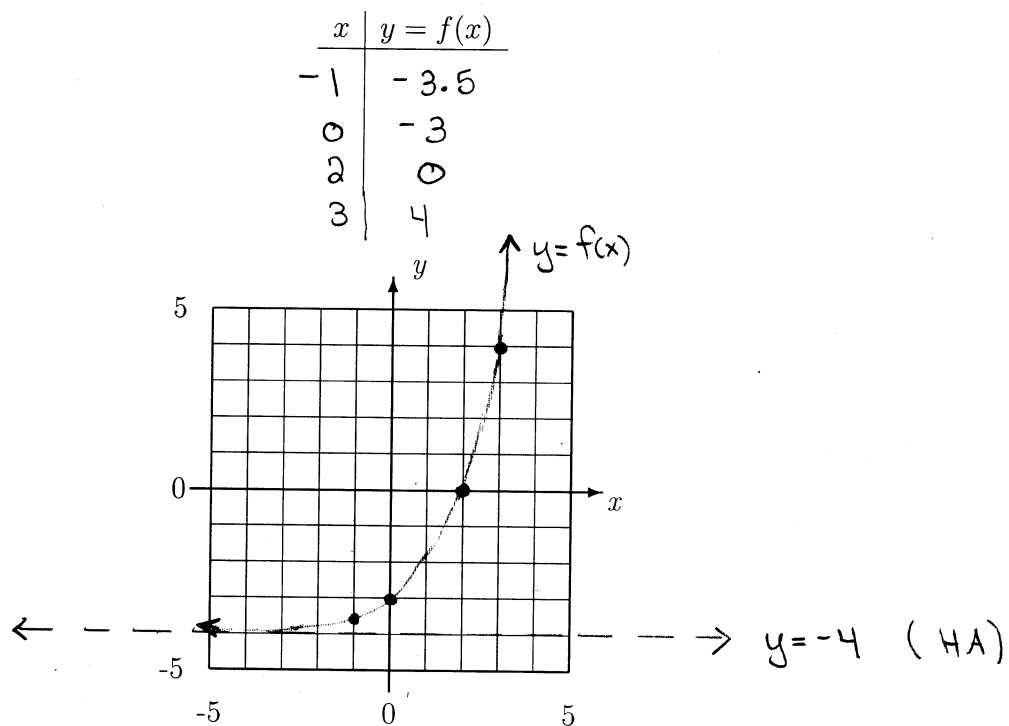
$$A(t) = 2500 (1.055)^t$$

- (b) How much is in the account after 30 years?

$$A(30) = 2500 (1.055)^{30}$$

$$= 12459.88$$

8. (8 points) Make a table of values showing 4 points on the graph of  $f(x) = 2^x - 4$ . Then sketch the graph of  $f$ . Label any asymptotes.



9. (9 points) Solve for  $x$ .

(a)  $2^{x+1} = 64^3$

$$2^{x+1} = (2^6)^3 = 2^{18} \Rightarrow x+1 = 18 \Rightarrow \boxed{x = 17}$$

(b)  $4^{2x-5} = 8^{x/2}$

$$(2^2)^{2x-5} = (2^3)^{x/2} \Rightarrow 4x-10 = \frac{3x}{2} \Rightarrow 2.5x = 10 \Rightarrow \boxed{x = 4}$$

(c)  $4^{4x-7} = \frac{1}{64}$

$$4^{4x-7} = 4^{-3} \Rightarrow 4x-7 = -3 \Rightarrow \boxed{x = 1}$$

10. (8 points) Consider the function  $f(x) = 1 + \log_2(x+4)$ . Complete the following table, plot the points, and sketch the graph, clearly labeling any asymptotes.

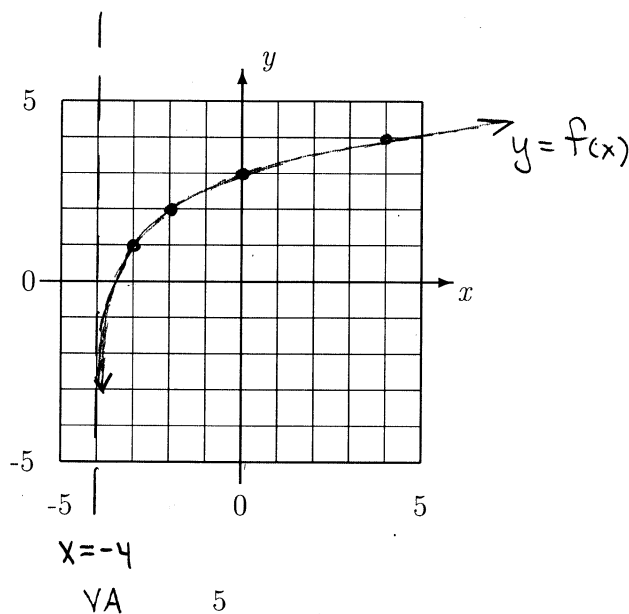
$x$	$y = f(x)$
-3	1
-2	2
0	3
4	4

$$1 + \log_2 1 = 1$$

$$1 + \log_2 2 = 2$$

$$1 + \log_2 4 = 3$$

$$1 + \log_2 8 = 4$$



11. (6 points) Without using a calculator, find the exact value of each expression. Show work or explain to receive full credit.

(a)  $\log_3 27 = \boxed{3}$

BECAUSE  $3^3 = 27$

(b)  $\log_{10} 0.000001 = \log_{10} 10^{-6} = \boxed{-6}$

(c)  $\log_a (a^3)^4 = \log_a a^{12} = \boxed{12}$

12. (3 points) Solve for  $x$ :  $\log_2(3x - 1) = 3$

$$\begin{aligned} 2^3 &= 3x - 1 \Rightarrow 8 = 3x - 1 \\ &\Rightarrow \boxed{x = 3} \end{aligned}$$

13. (2 points) Use your calculator to find the value of  $\log_{17} \pi$ . Round your answer to the nearest thousandth.

$$\frac{\ln \pi}{\ln 17} \approx \boxed{0.404}$$

14. (6 points) Use the properties of logarithms (i.e., the log laws) to condense each expression as much as possible, writing each answer as a single term with a coefficient of 1.

$$(a) \frac{1}{2} \ln x + 4 \ln y - 5 \ln z = \ln \left( \frac{\sqrt{x} y^4}{z^5} \right)$$

$$(b) \ln(x^3 y^2) - 2 \ln x - \ln y = \ln \left( \frac{x^3 y^2}{x^2 y} \right) = \ln(xy)$$

15. (6 points) Use the properties of logarithms (i.e., the log laws) to expand each expression as much as possible.

$$(a) \ln \left( \frac{x^2 y}{5} \right) = \ln x^2 + \ln y - \ln 5 = 2 \ln x + \ln y - \ln 5$$

$$(b) \log_{10}(\log_{10}(100x^3)) = \log_{10}(\overbrace{\log(100)}^{10^2=100} + \log x^3) \\ = \log(2 + 3 \log x)$$

16. (4 points) Draw a rough sketch the graph of  $y = a^x$ , where  $0 < a < 1$ .

