

Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) A nationally administered test has a mean of 500 and a standard deviation of 100. If your standardized score (z-score) on the test was 1.8, what was your actual test score?

$$1.8 = \frac{x - 500}{100} \Rightarrow 180 = x - 500$$
$$\Rightarrow \boxed{x = 680}$$

2. (5 points) Compute z-scores to determine which x-value has the higher relative position in its data set.

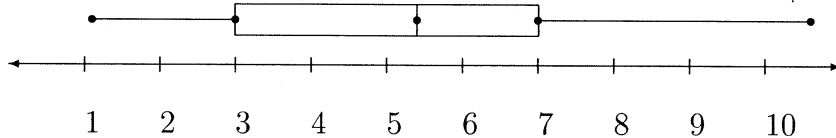
$x = 85$, where mean = 72 and standard deviation = 8
 $x = 93$, where mean = 87 and standard deviation = 5

$$z_{85} = \frac{85 - 72}{8} = 1.625$$

$$z_{93} = \frac{93 - 87}{5} = 1.2$$

85 HAS THE HIGHER
RELATIVE POSITION.

3. (6 points) The boxplot shown below describes a certain collection of data. Find approximate values for the median, first and third quartiles, and the interquartile range. Based on your approximations, what would be the cutoff values for outliers?



$$Q_1 \approx 3, \text{ MEDIAN} \approx 5.25,$$

$$Q_3 \approx 7, \text{ IQR} \approx 7 - 3 = 4$$

CUTOFFS FOR OUTLIERS
ARE -3 AND 13.

$$1.5 \times \text{IQR} = 6$$

$$Q_1 - 6 = -3$$

$$Q_3 + 6 = 13$$

4. (9 points) Refer to the ammonium ion concentration data.

(a) Find the percentile for the value 3.0.

$$\frac{12}{50} \times 100 = \boxed{24\%}$$

3.0 IS THE 24TH PERCENTILE

(b) Find the value of the 90th percentile.

$$\frac{l}{50} \times 100 = 90 \Rightarrow l = 45$$

$$\Rightarrow 90^{\text{TH}} \text{ PERCENTILE} = \frac{45^{\text{TH}} + 46^{\text{TH}}}{2} = \frac{5.5 + 5.6}{2} = \boxed{5.55}$$

(c) Find the value of the 65th percentile.

$$\frac{l}{50} \times 100 = 65 \Rightarrow l = 32.5$$

$$\Rightarrow 65^{\text{TH}} \text{ PERCENTILE} = 33^{\text{RD}} \text{ NUM} = \boxed{4.4}$$

5. (4 points) When Fred went to work at the liquor store, he wondered how good he was at guessing customers' ages. For the first 43 customers, Fred guessed correctly 29 times.

(a) Estimate the probability that Fred guesses correctly.

$$\frac{29}{43} \approx 67\%$$

(b) Is your estimate a theoretical, experimental, geometric, or subjective probability?

EXPERIMENTAL - HE ACTUALLY DID
TRIALS OF THE EXPERIMENT!

6. (10 points) Refer to the ammonium ion concentration data.

(a) Determine the three quartiles.

$$M_{\text{EDIAN}} = Q_2 = \frac{3.9 + 4.0}{2} = 3.95$$

$$Q_1 = 13^{\text{TH}} \text{ VALUE} = 3.0$$

$$Q_3 = 13^{\text{TH}} \text{ VALUE IN UPPER HALF} = 4.7$$

(b) Determine the interquartile range.

$$IQR = 4.7 - 3.0 = 1.7$$

(c) Determine the cut-off values for any outliers.

$$1.5 \times IQR = 2.55$$

$$Q_1 - 1.5 \times IQR = 0.45$$

$$Q_3 + 1.5 \times IQR = 7.25$$

(d) Identify the outliers (if any).

13.3 IS THE ONLY OUTLIER

(e) On the attached graph paper, construct the modified boxplot.

SEE ATTACHED SHEET.

7. (9 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box. The outcomes are recorded as ordered pairs of letters such as (a, b) .

a	a	a	b	b	c
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a	a	b	b	b	b
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- (a) Find the sample space for this experiment.

$$\{ (a, a), (a, b), (b, a), (b, b), \\ (c, a), (c, b), (c, c) \}$$

- (b) Is each outcome in your sample space equally likely? Explain.

No, (c, c) IS FAR LESS LIKELY THAN (a, a)
or (b, b) .

- (c) Let X be the event of drawing the letter b from the first box. What is \bar{X} ?

$$\begin{aligned} \bar{X} &= \text{NOT } b \text{ FROM } 1^{\text{ST}} \text{ BOX} \\ &= \{ (a, a), (a, b), (c, a), (c, b), (c, c) \} \end{aligned}$$

- (d) Let Y be the event of drawing the letter c from the second box. What is $X \cup Y$?

$$\begin{aligned} X \text{ OR } Y &= b \text{ FROM } 1^{\text{ST}} \text{ OR } c \text{ FROM } 2^{\text{ND}} = \\ &= \{ (b, a), (b, b), (c, c) \} \end{aligned}$$

- (e) What is $X \cap Y$?

$$\begin{aligned} X \text{ AND } Y &= b \text{ FROM } 1^{\text{ST}} \text{ AND } c \text{ FROM } 2^{\text{ND}} \\ &= \{ \} \quad \text{NO SUCH OUTCOMES.} \\ &\quad \text{IMPOSSIBLE EVENT.} \end{aligned}$$

8. (4 points) Given the probabilities $P(A) = 0.43$ and $P(B) = 0.59$, Sally concluded that $P(A \cup B) = 0.43 + 0.59 = 1.02$. What are two things you would say to Sally?

① SALLY HAS GOT TO BE WRONG BECAUSE A
PROBABILITY CANNOT EXCEED 1.

② SALLY NEEDS TO USE THE FORMULA:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

9. (3 points) If the odds against Big Joe Smasher winning the fight are 9 to 4, what is the probability of his winning?

$$\text{ODDS AGAINST} = \frac{9}{4}$$

$$\Rightarrow \text{ODDS IN FAVOR} = \frac{4}{9}$$

$$\Rightarrow \text{PROBABILITY} = \frac{4}{4+9} = \boxed{\frac{4}{13}}$$

10. (4 points) Three lower case letters of the English alphabet are selected without replacement. How many different 3-letter permutations are possible?

$$26 \cdot 25 \cdot 24 = \boxed{15,600}$$

11. (4 points) A scientist is labelling his samples with 7-letter sequences consisting of 4-X's, 2-Y's and 1-Z (XYYXZXX, for example). How many different sequences are possible?

$$\frac{7!}{4! \cdot 2! \cdot 1!} = \boxed{105}$$

12. (10 points) Suppose A and B are events such that $P(A) = 0.52$, $P(\bar{B}) = 0.36$, and $P(A \cup B) = 0.68$.

(a) Compute $P(B)$.

$$1 - 0.36 = \boxed{0.64}$$

(b) Compute $P(A \cap B)$.

$$0.68 = 0.52 + 0.64 - P(A \cap B)$$
$$\Rightarrow \boxed{P(A \cap B) = 0.48}$$

(c) Compute $P(A|B)$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.48}{0.64} = \boxed{0.75}$$

(d) Are A and B independent? Explain.

$$\text{No, } P(A|B) = 0.75 \neq 0.52 = P(A)$$

(e) What are the odds in favor of A ?

$$\frac{P(A)}{P(\bar{A})} = \frac{0.52}{1-0.52} = \boxed{\frac{52}{48} = \frac{13}{12}}$$

13. (6 points) There are 50 US Senators. A four person committee is to be formed.

(a) How many 4-person committees are possible?

$${}_{50}C_4 = \frac{50!}{46!4!} = \boxed{230,300}$$

(b) How many of those 4-person committees include Senator Dick Durbin?

Just choosing 3 people from 49.

$${}_{49}C_3 = \boxed{18,424}$$

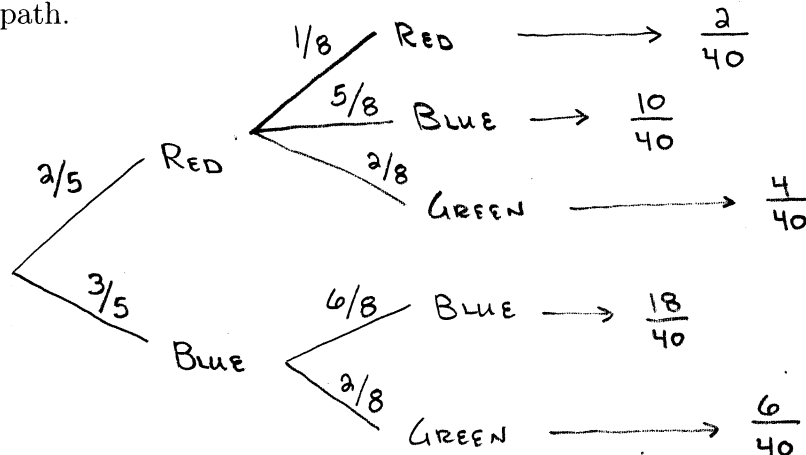
14. (6 points) A fair, six-sided die is rolled three times. What is the probability that at least one 5 is rolled?

$$\text{PROB THAT NO 5'S ARE ROLLED} = \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} = \frac{125}{216}$$

$$\text{PROB THAT AT LEAST ONE 5 IS ROLLED} = 1 - \frac{125}{216} = \boxed{\frac{91}{216}}$$

15. (10 points) Box One contains 2 red marbles and 3 blue marbles. Box Two contains 5 blue marbles and 2 green marbles. A marble is selected at random from Box One and placed into Box Two. Then a marble is selected at random from Box Two.

- (a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



- (b) What is the probability of drawing a blue marble from Box Two?

$$\frac{10}{40} + \frac{18}{40} = \boxed{\frac{28}{40}}$$

- (c) What is the probability of drawing a blue marble from Box One or a green marble from Box Two?

$$\frac{6}{40} + \frac{18}{40} + \frac{4}{40} = \boxed{\frac{28}{40}}$$

- (d) What is the probability of drawing a blue marble from Box Two given that a blue marble was drawn from Box One?

$$\boxed{\frac{6}{8}}$$

16. (6 points) The numbers of students at a certain college are described in the table below.

	Female	Male	
Part-time	2112	1408	3520
Full-time	1746	1164	2910
	3858	2572	6430

← TOTAL OF
6430

A college student is selected at random.

(a) What is the probability that the student is a female?

$$\frac{3858}{6430}$$

(b) What is the probability that the student is a part-time student?

$$\frac{3520}{6430}$$

(c) What is the probability that the student is a female, part-time student?

$$\frac{2112}{6430}$$

(d) What is the probability that the student is a female given that the student is part-time?

$$\frac{2112}{3520}$$

(e) Are being a female student and being a part-time student independent events? Use some of your results from above to support your answer.

$$\begin{aligned} \text{PROB OF FEMALE GIVEN PART-TIME} &= \frac{2112}{3520} = 0.6 \\ \text{PROB OF FEMALE} &= \frac{3858}{6430} = 0.6 \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{PROB OF FEMALE GIVEN PART-TIME} \\ \text{PROB OF FEMALE} \end{aligned}} \right\} \text{YES!}$$

17. (Extra Credit—5 points) In a group of ten iPods, there are two that do not work. From the group of ten, three iPods are selected at random. Let the random variable x represent the number of iPods in the sample that do not work. Find the probability distribution for x . Then find the mean and standard deviation for the variable x .

X TAKES THREE POSSIBLE VALUES: 0, 1, 2

$$P(x=0) = \frac{{}^8C_3}{{}^{10}C_3} = \frac{56}{120} = \boxed{\frac{7}{15}}$$

$$P(x=1) = \frac{{}^8C_2 \cdot {}^2C_1}{{}^{10}C_3} = \frac{28 \cdot 2}{120} = \frac{56}{120} = \boxed{\frac{7}{15}}$$

$$P(x=2) = \frac{{}^8C_1 \cdot {}^2C_2}{{}^{10}C_3} = \frac{8}{120} = \boxed{\frac{1}{15}}$$

You could
also use
a tree to
get these.

x	$P(x)$
0	$\frac{7}{15}$
1	$\frac{7}{15}$
2	$\frac{1}{15}$

$$\mu = 0\left(\frac{7}{15}\right) + 1\left(\frac{7}{15}\right) + 2\left(\frac{1}{15}\right) = \frac{9}{15} = \boxed{0.6}$$

$$\sigma^2 = \left[0\left(\frac{7}{15}\right) + 1\left(\frac{7}{15}\right) + 4\left(\frac{1}{15}\right) \right] - \left(\frac{9}{15}\right)^2 = \frac{11}{15} - \frac{81}{225} = \frac{28}{75}$$

$$\sigma = \sqrt{\frac{28}{75}} \approx \boxed{0.611}$$

Concentration of Ammonium Ions in 50 Samples

(In Percentages)

1.4	2.3	2.4	2.6	2.6	2.7	2.7	2.8	2.8	2.9
2.9	2.9	3.0	3.1	3.1	3.2	3.3	3.4	3.5	3.5
3.6	3.7	3.7	3.9	3.9	4.0	4.0	4.1	4.1	4.2
4.2	4.2	4.4	4.5	4.6	4.6	4.6	4.7	4.8	4.8
4.8	4.9	5.2	5.2	5.5	5.6	5.7	6.5	7.0	13.3

#6e Boxplot

