Math 153 - Test 3a
November 20, 2012

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. YOU MUST WORK INDIVIDUALLY ON THIS EXAM.

1. (6 points) Consider the table shown below.

x	$P(x)$
3	0.003
4	0.012
5	0.037
6	0.325
7	0.028
8	0.462
9	0.084
10	0.049

(a) Does the table describe a probability distribution? Explain.

YES, $0.003 + 0.012 + 0.037 + 0.325 + 0.028 + 0.462 + 0.084 + 0.049 = 1$

(b) What are unusually small values of x?

3 & 4 SINCE $P(x \leq 4) < 0.05$

BUT $P(x \leq 5) > 0.05$

(c) What are unusually large values of x?

9 SINCE $P(x \geq 10) < 0.05$

BUT $P(x \geq 9) > 0.05$

2. (3 points) Sam likes the cafeteria's grilled cheese sandwiches, but he finds they are overcooked 32% of the time. In his next ten visits to the cafeteria, what is the probability that at least two of his sandwiches will be overcooked?

BINOMIAL
$N = 10$
$p = 0.32$
$X \geq 2$

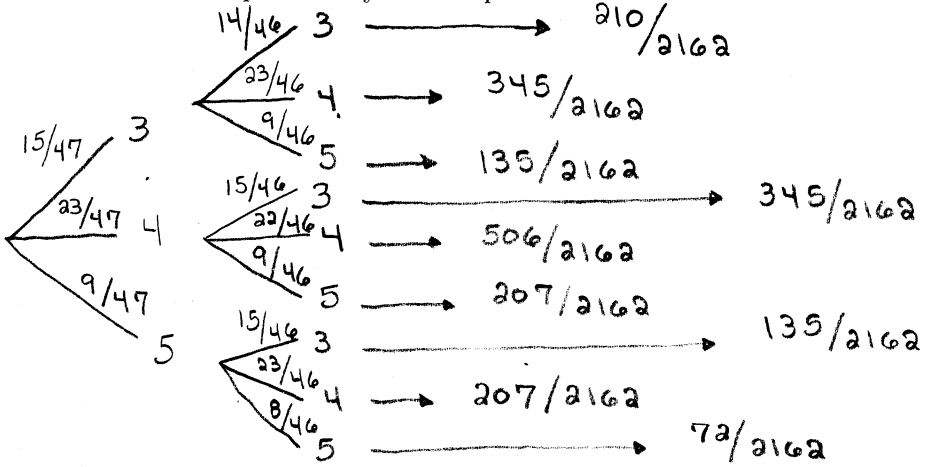
$P(x \geq 2) = 1 - P(x \leq 1)$

$= 1 - \text{binomialcdf}(10, 0.32, 1)$

$= 0.879$

3. In a certain neighborhood, there are 15 three-person households, 23 four-person households, and 9 five-person households. Two households are picked at random without replacement. Let the random variable x represent the total number of people in the two households.

(a) (5 points) Sketch the probability tree associated with the selection of the two households. Find the probability of each path.



(b) (2 points) What are the possible values of the random variable x?

6, 7, 8, 9, 10

(c) (4 points) Determine the probability distribution for the random variable x. Give your distribution in the form of a table.

x	P(x)
6	210/2162
7	690/2162
8	776/2162
9	414/2162
10	72/2162

(d) (3 points) Find the mean value of x.

$$\mu = (6)\left(\frac{210}{2162}\right) + (7)\left(\frac{690}{2162}\right) + (8)\left(\frac{776}{2162}\right) + (9)\left(\frac{414}{2162}\right) + (10)\left(\frac{72}{2162}\right)$$

$$= \frac{16744}{2162} \approx 7.74$$

(e) (3 points) Find the standard deviation in the values of x.

$$\sigma^2 = (36)\left(\frac{210}{2162}\right) + (49)\left(\frac{690}{2162}\right) + (64)\left(\frac{776}{2162}\right) + (81)\left(\frac{414}{2162}\right) + (100)\left(\frac{72}{2162}\right) - \left(\frac{16744}{2162}\right)^2$$

$$= \frac{131768}{2162} - \left(\frac{16744}{2162}\right)^2 \approx 0.96718956$$

$$\sigma \approx 0.983$$

4. (6 points) A candidate for a state senate seat found that in a certain region 74% of the registered voters recognized her name. Twenty registered voters are selected at random. Let x represent the number of voters in the sample who recognize the candidate's name.

BINOMIAL : $N = 20$, $p = 0.74$

(a) What is the mean value of x?

$$\mu = 20 \cdot (0.74) = 14.80$$

(b) What is the standard deviation in the values of x?

$$\sigma = \sqrt{20(0.74)(0.26)} = 1.96$$

(c) What are the cutoff values for the unusually small and unusually large values of x?

$$\mu - 2\sigma = 10.88$$

$$\mu + 2\sigma = 18.72$$

5. (6 points) In the state of Illinois from 1956 to 2004, there were an average of 35.2 tornadoes per year.

(a) In any given year, what is the probability that Illinois has 40 or more tornadoes?

$$P(x \geq 40) = 1 - P(x \leq 39)$$

$$= 1 - \text{poissoncdf}(35.2, 39) = 0.2302 \approx 23\%$$

(b) In 2003, Illinois experienced 120 tornadoes. Is this an unusually large number of tornadoes? Explain.

$\mu = 35.2$

$\sigma = \sqrt{\mu} = 5.93$

$\mu + 2\sigma = 47.06$

SINCE $120 > \mu + 2\sigma$, IT IS UNUSUALLY LARGE.

(c) In any given year, what is an unusually small number of tornadoes?

$$\mu - 2\sigma = 23.34 \approx 23$$

23 OR FEWER

6. (6 points) The American Automobile Association (AAA) reports that the response times for vehicle emergency calls are normally distributed with a mean of 25 minutes and a standard deviation of 4.5 minutes.

   (a) What is the probability that a response time will be 20 minutes or less?

normalcdf(-99999, 20, 25, 4.5)

= 0.13326 ≈ 13.3%

   (b) In a sample of 80 calls, about how many will have response times exceeding 32 minutes?

80 x normalcdf(32, 99999, 25, 4.5)

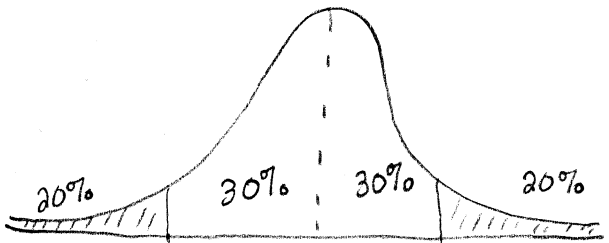
= 4.79 ⇒ ABOUT 5

   (c) What response time is at the 75th percentile?

invNorm (0.75, 25, 4.5) = 28.035

7. (6 points) An automobile dealer finds that used car prices are normally distributed with mean \$8256 and standard deviation \$1150. The dealer decides to sell cars that appeal to the middle 60% of the market in terms of price. Find the minimum and maximum prices of the cars the dealer will sell.

DEALER WILL SELL CARS BETWEEN \$7288.14 AND \$9223.84.



Z = invNorm (0.80, 8256, 1150)

= 9223.84

Z = invNorm (0.20, 8256, 1150)

= 7288.14

8. (6 points) The mean yearly Medicare Hospital Insurance benefit per person is \$4064. Suppose the benefits are normally distributed with standard deviation \$460. A random sample of 20 patients is obtained.

   (a) What is the probability that the sample mean is less than \$3800.

$$\text{normalcdf}(-99999, 3800, 4064, 460/\sqrt{20}) = 0.00513$$

$$\approx \underline{0.5\%}$$

   (b) What is the probability that the sample mean is more than \$4100.

$$\text{normalcdf}(4100, 99999, 4064, 460/\sqrt{20}) = 0.3631719$$

$$\approx \underline{36.3\%}$$

9. (9 points) In a survey of 150 senior executives, 47% said the most common job interview mistake is to have little or no knowledge of the company.

   (a) Construct a 99% confidence interval estimate for the proportion of all senior executives who have the same opinion.

Could also use 1-PropZInt with $x = 70$ or 71

$z_{\alpha/2} = 2.5758$

$\hat{p} = 0.47$

$\hat{q} = 0.53$

$N = 150$

$E = 0.105$

$(\hat{p} - E, \hat{p} + E) = \underline{(0.365, 0.575)}$

   (b) Is it possible that 60% of all senior executives believe that the most common interview mistake is to have little or no company knowledge? Explain.

Since 0.60 is not in the 99% confidence interval, it is not very likely.

   (c) Using the point estimate $\hat{p} = 0.47$, determine the sample size required to have a margin of error of ± 0.04.

$$N = \frac{(2.5758)^2 (0.47)(0.53)}{(0.04)^2} = 1032.947$$

$$\approx \underline{1033}$$

10. (10 points) The ages, in years, of the four U.S. presidents when they were assassinated in office are 56 (Lincoln), 49 (Garfield), 58 (McKinley), and 46 (Kennedy).

(a) Two of the ages are selected at random with replacement. List all possible samples and find the median of each sample.

(56, 56) – 56	(49, 56) - 52.5	(58, 56) - 57	(46, 56) - 51
(56, 49) - 52.5	(49, 49) - 49	(58, 49) - 53.5	(46, 49) - 47.5
(56, 58) - 57	(49, 58) - 53.5	(58, 58) - 58	(46, 58) - 52
(56, 46) - 51	(49, 46) - 47.5	(58, 46) - 52	(46, 46) - 46

(b) Summarize the sampling distribution in a probability distribution table.

MEDIAN, X	46	47.5	49	51	52	52.5	53.5	56	57	58
P(x)	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

(c) Find the mean of the sample medians.

$$46 + 2(47.5) + 49 + 2(51) + 2(52) + 2(52.5) + 2(53.5) + 56 + 2(57) + 58 = 836$$

$$\Rightarrow \text{MEAN} = \frac{836}{16} = \underline{52.25}$$

(d) Find the population median.

46, 49, 56, 58

↑

MEDIAN = 52.5

(e) Do the sample medians target the population median? Explain.

No, $52.25 \neq 52.5$

Math 153 - Test 3b
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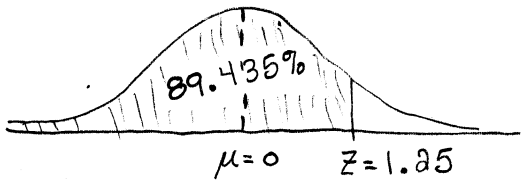
1. (3 points) Sketch the standard normal curve and shade the region under the curve associated with the probability $P(z < 1.25)$.



2. (3 points) A 90% confidence interval estimate for the mean number of farms per state is $(34.3, 52.7)$. Find the best estimate for the mean number of farms per state and the margin of error.

$\hat{p} - E = 34.3$

$\hat{p} + E = 52.7$

$2\hat{p} = 87$

$\hat{p} = \frac{87}{2} = 43.5$

$E = 52.7 - \hat{p} = 9.2$

3. (3 points) A pizza shop owner wishes to find the 95% confidence interval estimate of the actual mean cost of a plain cheese pizza. How large should the sample be if she wishes to be accurate to within \$0.15? A previous study showed that the standard deviation of the price was \$0.26.

95% C-Level $\Rightarrow$ $\alpha = 0.05$

$\Rightarrow$ $\alpha/2 = 0.025$

$z_{\alpha/2} = 1.96$

$E = 0.15$

$\sigma = 0.26$

$N = \left(\frac{1.96 \cdot 0.26}{0.15}\right)^2 = 11.54$

A sample size of 12 should do it.

4. (3 points) In a certain heavily polluted lake, 62% of the frogs have unusual deformities. In a random sample of 20 captured frogs, what is the probability that exactly 10 have unusual deformities?

$N = 20$

$p = 0.62$

$q = 0.38$

$k = 10$

$P(x = 10) = \text{binomialpdf}(20, 0.62, 10)$

$= 0.09735$

$= 9.735\%$

5. (4 points) Teacher's salaries in North Dakota are normally distributed with mean \$35441 and standard deviation \$5100. In a sample of 500 teachers, about how many have salaries between \$30000 and \$50000?

$500 \times \text{normalcdf}(30000, 50000, 35441, 5100)$

$= 427.4145802$

$\approx \underline{427}$

6. (3 points) Heights of men are normally distributed with mean 69 in and standard deviation 2.8 in. A contractor wishes to determine the height of a basement ceiling that allows 99.5% of all men to stand without crouching. What would that height be?

$\text{invNorm}(0.995, 69, 2.8) = 76.21 \text{ IN}$

7. (6 points) Five \$1 bills, ten \$5 bills, and seven \$10 bills are placed into a box, and one bill is selected at random. Let x be the value of the selected bill.

(a) Construct the probability distribution for the random variable x.

x	$P(x)$
1	$5/22$
5	$10/22$
10	$7/22$

(b) Find the expected value of x.

$$\mu = (1)\left(\frac{5}{22}\right) + (5)\left(\frac{10}{22}\right) + (10)\left(\frac{7}{22}\right)$$

$$= \frac{5}{22} + \frac{50}{22} + \frac{70}{22} = \frac{125}{22} \approx \$5.68$$

(c) Find the standard deviation in the values of x.

$$\sigma^2 = (1)\left(\frac{5}{22}\right) + (25)\left(\frac{10}{22}\right) + (100)\left(\frac{7}{22}\right) - \left(\frac{125}{22}\right)^2$$

$$\approx 11.126$$

$$\sigma \approx \$3.34$$