Math 153 - Final Exam

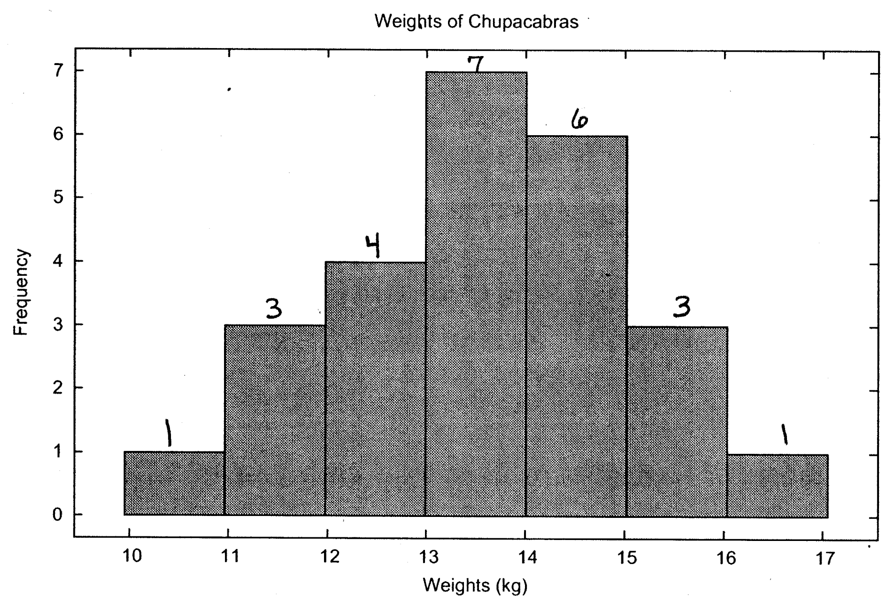
December 13, 2012

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) An eccentric Texan named Jorge Bosh claims he has captured and is raising a number of Chupacabras. The histogram below shows the weights of his Chupacabras in kilograms.



(a) How many Chupacabras does Jorge have in captivity?

$1+3+4+7+6+3+1 = \boxed{25}$

(b) If the histogram was changed to a relative frequency histogram, what would be the height of the third bar?

$\frac{4}{25} = \boxed{0.16}$

(c) Are the weights of his Chupacabras normally distributed? Explain.

YES. IT LOOKS LIKE THEY ARE APPROXIMATELY NORMAL.

THE HISTOGRAM IS ROUGHLY SYMMETRIC WITH A SMOOTH, BELL-CURVE OUTLINE.

2. (4 points) Refer to Problem 1. Use class midpoints to compute the mean weight of Jorge's Chupacabras.

$$\frac{(10.5)(1)+(11.5)(3)+(12.5)(4)+(13.5)(7)+(14.5)(6)+(15.5)(3)+(16.5)(1)}{25}$$

$$= \boxed{13.58 \text{ kg}}$$

3. (12 points) In a large sample of men, the mean height was 70.2 in with a standard deviation of 2.8 in. The mean weight was 172 lbs with a standard deviation of 29 lbs.

(a) Compute z-scores to determine which is "relatively" greater, 75.7 in or 231 lbs.

HEIGHT:

$$z = \frac{75.7 - 70.2}{2.8} \approx \boxed{1.964}$$

WEIGHT:

$$z = \frac{231 - 172}{29} \approx \boxed{2.034}$$

THE WEIGHT IS GREATER.

(b) Compute the coefficient of variation for the heights and the weights. Is there more spread in the heights or weights?

HEIGHT:

$$CV = \frac{2.8}{70.2} \approx \boxed{4\%}$$

WEIGHT:

$$CV = \frac{29}{172} \approx \boxed{17\%}$$

MORE SPREAD IN WEIGHTS

(c) Based on the sample data, at what weight should a man be considered unusually light?

$$172 - 2(29) = \boxed{114 \text{ lbs}}$$

4. (12 points) The numbers of tornadoes in Illinois for each year from 1990 to 1999 are shown below.

$$50, 32, 23, 34, 20, 76, 62, 29, 99, 64$$

(a) Find the range and the sample standard deviation.

RANGE = $99 - 20 = 79$

$S_x \approx 25.94$ (FROM CALCULATOR)

(b) Find the median, quartiles, and the interquartile range.

$Q_1 = 29$

MED = $Q_2 = 42$ FROM CALCULATOR

$Q_3 = 64$

$IQR = 64 - 29 = 35$

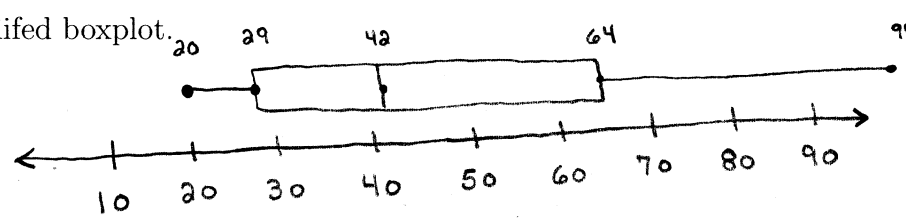
(c) Compute the cut-off values for outliers.

$Q_1 - 1.5 * IQR = -23.5$

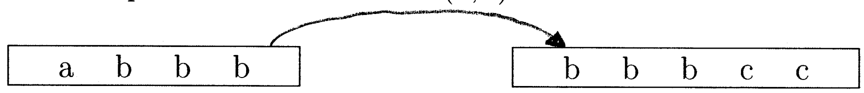
$Q_3 + 1.5 * IQR = 116.5$

THERE ARE NO OUTLIERS IN THE DATA.

(d) Sketch the modifed boxplot.



5. (10 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box. The outcomes are recorded as ordered pairs of letters such as (a, b).



(a) Find the sample space for this experiment.

$\{(a,a), (a,b), (a,c), (b,b), (b,c)\}$

(b) Is each outcome in your sample space equally likely? Explain.

NO, (a,a) IS RATHER UNLIKELY,

WHILE (b,b) IS PRETTY LIKELY.

(c) Let X be the event of drawing the letter a from the first box. What is $\overline{X}$?

$\{(b,b), (b,c)\}$

(d) Let Y be the event of drawing the letter c from the second box. What is $X \cup Y$?

$\{(a,a), (a,b), (a,c), (b,c)\}$

(e) What is $X \cap Y$?

$\{(a,c)\}$

6. (10 points) Suppose A and B are events such that $P(\overline{A}) = 0.52$, $P(B) = 0.55$, and $P(A \cup B) = 0.766$.

(a) Compute $P(A)$.

$$1 - P(\bar{A}) = \boxed{0.48}$$

(b) Compute $P(A \cap B)$.

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \boxed{0.264}$$

(c) Compute $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \boxed{0.55}$$

(d) Are A and B independent? Explain.

YES, BECAUSE $P(B|A) = P(B)$

(e) What are the odds in favor of A?

$$\frac{P(A)}{P(\bar{A})} = \frac{0.48}{0.52} = \frac{48}{52} = \boxed{\frac{12}{13}}$$

7. (9 points) A certain sales firm receives, on average, 132 calls per day.

(a) On any given day, what is the probability of the firm receiving 145 or more calls?

POISSON

$$P(x \geq 145) = 1 - P(x \leq 144)$$

$$= 1 - \text{poissoncdf}(132, 144) \approx \boxed{0.1388} = 13.88\%$$

(b) On Tuesday the firm received 153 calls. Is that an unusually large number of calls?

$$\mu + 2\sigma = 132 + 2\sqrt{132} \approx 154.98$$

No, 153 IS NOT "UNUSUALLY" LARGE.

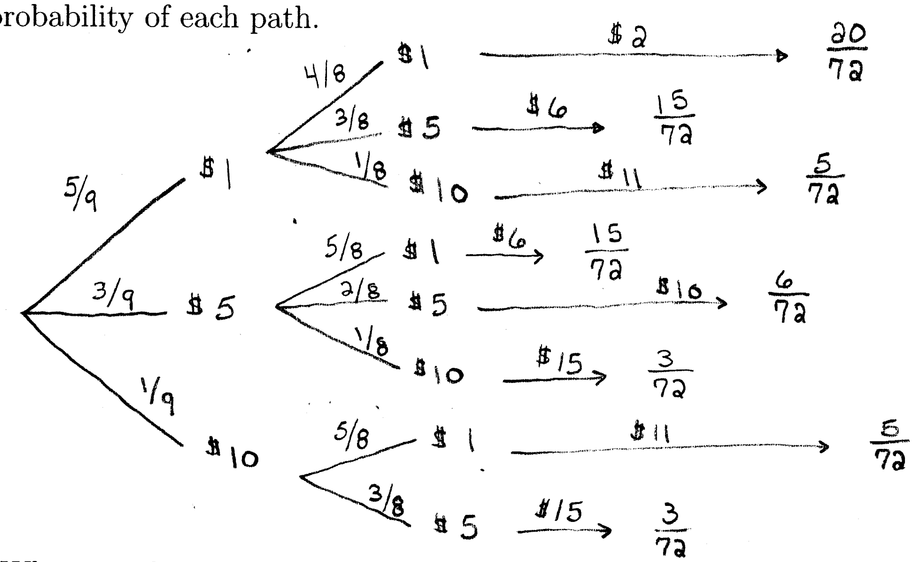
(c) On any given day, what would be an unusually small number of calls?

$$\mu - 2\sigma = 132 - 2\sqrt{132} \approx 109.02$$

$\boxed{109 \text{ OR FEWER}}$

8. A box contains five \$1-bills, three \$5-bills, and one \$10-bill. Two bills are selected at random without replacement. Let the random variable x represent the total amount of money selected.

(a) (6 points) Sketch the probability tree associated with the selection of the two bills. Find the probability of each path.



(b) (2 points) What are the possible values of the random variable x?

2, 6, 10, 11, 15

(c) (4 points) Determine the probability distribution for the random variable x. Give your distribution in the form of a table.

X	2	6	10	11	15
P(x)	$\frac{20}{72}$	$\frac{30}{72}$	$\frac{6}{72}$	$\frac{10}{72}$	$\frac{6}{72}$

(d) (4 points) Find the mean value of x.

$$2\left(\frac{20}{72}\right) + 6\left(\frac{30}{72}\right) + 10\left(\frac{6}{72}\right) + 11\left(\frac{10}{72}\right) + 15\left(\frac{6}{72}\right) = \frac{480}{72} \approx \boxed{\$6.67}$$

(e) (4 points) Find the standard deviation in the values of x.

$$\sigma^2 = 4\left(\frac{20}{72}\right) + 36\left(\frac{30}{72}\right) + 100\left(\frac{6}{72}\right) + 121\left(\frac{10}{72}\right) + 225\left(\frac{6}{72}\right) - \left(\frac{480}{72}\right)^2 = 15.\overline{5}$$

$$\sigma = \sqrt{15.\overline{5}} \approx \boxed{3.94}$$

9. (9 points) In the past year, 27% of small businesses in Illinois have eliminated jobs. A random sample of 15 Illinois small businesses is selected.

(a) What is the probability that at least 8 of those businesses have eliminated jobs?

BINOMIAL

$$P(x \geq 8) = 1 - P(x \leq 7)$$

$$= 1 - \text{binomialcdf}(15, 0.27, 7) \approx \boxed{0.0274} \approx 3\%$$

(b) Suppose that only one of those businesses actually eliminated jobs. Is that an unusually small number?

$$\mu - 2\sigma = 0.27(15) - 2\sqrt{0.27 \cdot 0.73 \cdot 15}$$

$$\approx 0.6111 \Rightarrow$$ 1 IS NOT UNUSUALLY SMALL.

(c) What is the probability that fewer than 3 of those businesses have eliminiated jobs?

$$\text{binomialcdf}(15, 0.27, 2) \approx \boxed{0.1863} = 18.63\%$$

10. (10 points) The mean yearly Medicare spending per beneficiary is \$5694. Suppose the spendings are normally distributed with standard deviation \$612. A random sample of 15 patients is obtained.

(a) What is the standard deviation of the sampling distribution?

$$S_{\bar{x}} = \frac{612}{\sqrt{15}} \approx 158.02$$

(b) What is the probability that the sample mean is less than \$4800?

$$\text{normalcdf}(-99999, 4800, 5694, 612/\sqrt{15})$$

$$\approx 7.697 \times 10^{-9}$$

(c) What is the probability that the sample mean is more than \$6700?

$$\text{normalcdf}(6700, 99999, 5694, 612/\sqrt{15})$$

$$\approx 9.721 \times 10^{-11}$$

11. (10 points) The monthly amounts of paper waste generated by an American household are normally distributed with mean 28 lbs and standard deviation 2 lbs.

(a) What amount of paper waste is at the 75th percentile?

invNorm(0.75, 28, 2) $\approx \boxed{29.35 \text{ lbs}}$

(b) What is an unusally large amount of paper waste?

$28 + 2(2) = 32$ lbs

ANY MORE THAN 32 lbs.

(c) In a sample of 500 households, about how many would generate more than 31.5 lbs of paper waste?

500 * normalcdf(31.5, 99999, 28, 2) $\approx \boxed{20}$

12. (16 points) A large university reports that the mean salary of parents of an entering class is \$91,600. The university president randomly selects 28 families, and she finds the mean salary to be \$88,500 with a sample standard deviation of of \$9,915. Use the president's sample to test the university's reported claim at the level $\alpha = 0.10$.

(a) State the null and alternative hypotheses.

$H_0: \mu = 91600$

$H_1: \mu \neq 91600$

(b) What is the underlying sampling distribution?

STUDENT'S t (σ IS NOT KNOWN)

(c) Compute the test statistic.

$$t = \frac{88500 - 91600}{9915/\sqrt{28}} \approx -1.654$$

(d) Find the P-value and draw a conclusion about the university's claim.

$\underbrace{\boxed{\text{P-VALUE} \approx 0.1096}}_{\text{FROM CALCULATOR}}$

SINCE P-VALUE > 0.10, THERE IS NOT SUFFICIENT EVIDENCE TO REJECT THE CLAIM.

13. (12 points) The data below were obtained in a study of student success in Math 171.

Number of absences x	Final grade y (%)
6	82
2	86
15	43
9	74
12	58
5	90
8	78

(a) Compute the linear correlation coefficient, r, and use it to draw a conclusion about the strength of the linear relationship between x and y.

$\boxed{r \approx -0.944}$ (CALCULATOR)

SINCE r IS CLOSE TO -1,

THERE IS EVIDENCE OF A STRONG NEGATIVE LINEAR CORRELATION.

(b) Compute the corresponding P-value and draw a conclusion about the existence of a linear relationship.

$\boxed{\text{P-VALUE} \approx 0.0014}$ (CALCULATOR)

SINCE $0.0014 < 0.01$,

WE HAVE EVIDENCE FOR THE EXISTENCE OF A LINEAR RELATIONSHIP AT THE LEVEL $\alpha = 0.01$.

(c) Find the regression equation and use it to predict the final grade of a student with 3 absences.

$\boxed{\hat{y} = -3.622x + 102.493}$ (CALCULATOR)

WHEN $x = 3$, $\boxed{\hat{y} = 91.6\ \%}$

14. (10 points) A survey of 35 adults found that the mean age of a person's primary vehicle is 5.6 years. Assume that the standard deviation of the population is 0.8 years.

(a) Is the underlying sampling distribution normal or t? How do you know?

NORMAL, σ IS KNOWN.

(b) Construct a 95% confidence interval estimate for the mean age of all primary vehicles.

CALCULATOR, USING ZInterval

$\sigma = 0.8$

$\bar{X} = 5.6$

$N = 35$

$$\boxed{(5.335,\ 5.865)}$$

(c) Determine the sample size required to have a margin of error of ± 0.02 at the level $\alpha = 0.01$.

$$N = \left(\frac{z_{\alpha/2}\,\sigma}{E}\right)^2 = \left(\frac{2.5758 \cdot 0.8}{0.02}\right)^2 \approx \boxed{10,616}$$