Math 153 - Test 3

November 15, 2018

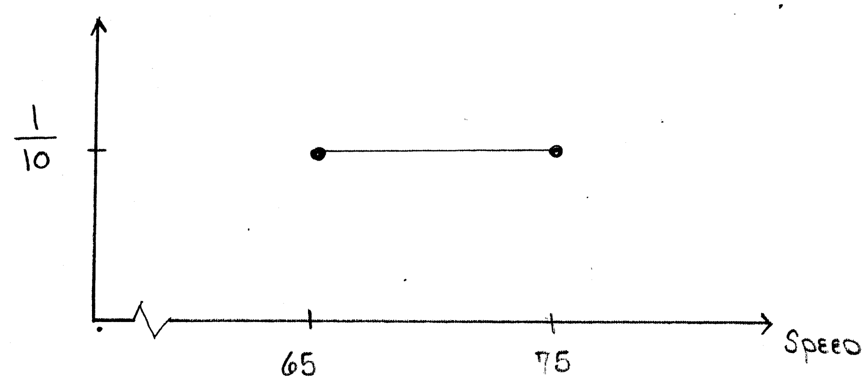
Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary. You may use your calculator for all statistical computations, but show how you use it.

1. (9 points) On a certain section of the local interstate highway, the speeds of cars are uniformly distributed between 65 mph and 75 mph.

   (a) Sketch the density curve for the probability distribution.



   (b) What is the probability that a random car is driving faster than 73 mph?

$$P(x > 73) = (75-73)\left(\frac{1}{10}\right) = \boxed{\frac{2}{10} = 0.2 = 20\%}$$

   (c) What speed is at the 65th percentile?

$$(X-65)\left(\frac{1}{10}\right) = 0.65 \Rightarrow X-65 = 6.5$$

$$X = 71.5$$

$$\boxed{71.5 \text{ mph}}$$

2. (4 points) The odds against the event B are $5 : 9$. What are the odds in favor of B? What is the probability of B?

$$\boxed{P(B) = \frac{9}{9+5} = \frac{9}{14}}$$

$$\boxed{\frac{9}{5}}$$

3. (4 points) Suppose that A, B, C, and D are mutually exclusive (disjoint) events that exhaust the sample space? If $P(A) = 5/8$ and $P(C) = 1/8$, then what is the probability of $B \cup D$? (Show work or explain.)

$$P(A \cup B \cup C \cup D) = 1 = P(A) + P(B) + P(C) + P(D)$$
$$= P(A) + P(C) + P(B \cup D)$$
$$= \frac{5}{8} + \frac{1}{8} + ? \Rightarrow P(B \cup D) = \frac{2}{8}$$

4. (10 points) American children are more engaged in sports than they were 15 years ago. Currently in the U.S., the probability that a randomly selected child is involved in school sports is 0.42. Ten children are selected at random.

BINOMIAL
$n = 10$
$p = 0.42$
$q = 0.58$

(a) What is the probability that exactly 5 of the children are involved in sports?

$$P(X=5) = \text{binompdf}(10, 0.42, 5)$$
$$\approx 0.2162 = 21.62\%$$

(b) What is the probability that more than 5 are involved in sports?

$$P(X>5) = 1 - P(X \leq 5)$$
$$= 1 - \text{binomcdf}(10, 0.042, 5)$$
$$\approx 0.2016 = 20.16\%$$

(c) In the sample of 10, what is the expected number of children involved in sports?

$$\mu = np = 4.2$$

(d) In the sample of 10, what are unusually small and large numbers of children involved in sports?

$$\sigma = \sqrt{npq} \approx 1.56$$

$$\mu - 2\sigma \approx 1.08$$
$$\mu + 2\sigma \approx 7.32$$
$\Rightarrow$

SMALL: 1 OR FEWER
LARGE: 8 OR MORE

5. (10 points) The probability distribution for the random variable x is shown below.

x	0	1	2	3	4	5
$P(x)$	0.12	0.03	0.74	0.07	0.01	0.03

(a) What two things about the table above show that it is a probability distribution?

① FOR EACH X, $0 \leq P(x) \leq 1$

② $\sum P(x) = 1$

(b) What is the mean value of x?

$$\mu = 0(0.12) + 1(0.03) + 2(0.74) + 3(0.07) + 4(0.01) + 5(0.03)$$

$$= \boxed{1.91}$$

(c) What is the standard deviation in x?

$$\sigma^2 = 0(0.12) + 1(0.03) + 4(0.74) + 9(0.07) + 16(0.01) + 25(0.03) - 1.91^2$$

$$= 0.8819 \Rightarrow \sigma = \sqrt{0.8819} \approx \boxed{0.939}$$

(d) Determine $P(x > 2)$.

$$0.07 + 0.01 + 0.03 = \boxed{0.11}$$

(e) Determine all unusual values of x.

NO UNUSUALLY SMALL VALUES: $P(x \leq 0) > 5\%$

4 & 5 ARE UNUSUALLY LARGE: $P(x \geq 4) = 0.04 < 5\%$

6. (3 points) A fun size bag of Skittles contains 13 candies, 6 of which happen to be yellow. (Therefore the probability of randomly selecting a yellow candy is 6/13.) Suppose 10 candies are selected at random and eaten. Let x represent the number of yellow candies in the sample. Are the values of x in a binomial distribution? Explain.

NO. THE TRIALS ARE NOT INDEPENDENT (WITHOUT REPLACEMENT), AND IT WOULD NOT BE APPROPRIATE TO USE THE BINOMIAL DIST. AS AN APPROX BECAUSE THE SAMPLE IS SO LARGE COMPARED TO THE POPULATION.

7. (8 points) Some adults are selected at random and asked whether they are right or left handed. The results are shown below.

	Female	Male	
Right handed	44	43	87
Left handed	4	9	13
	48	52	100

A person from the sample is selected at random.

(a) What is the probability that the person is a female?

$\frac{48}{100}$

(b) What is the probability that the person is a left-handed female?

$\frac{4}{100}$

(c) What is the probability that the person is a female given that the person is left handed?

$\frac{4}{13}$

(d) Are the events of selecting a female and selecting a left-handed person independent. Use some of your results from above to support your answer.

No, BECAUSE (a) $\neq$ (c).

8. (6 points) A letter is selected at random from the word *racecars*. Let A be the event of selecting a vowel, and let B be the event of selecting the letter e.

(a) Determine $P(B|A)$.

SELECT e FROM THE VOWELS

$\boxed{\frac{1}{3}}$

(b) Determine $P(A|B)$.

SELECT VOWEL FROM e'S

$\boxed{\frac{1}{1}}$

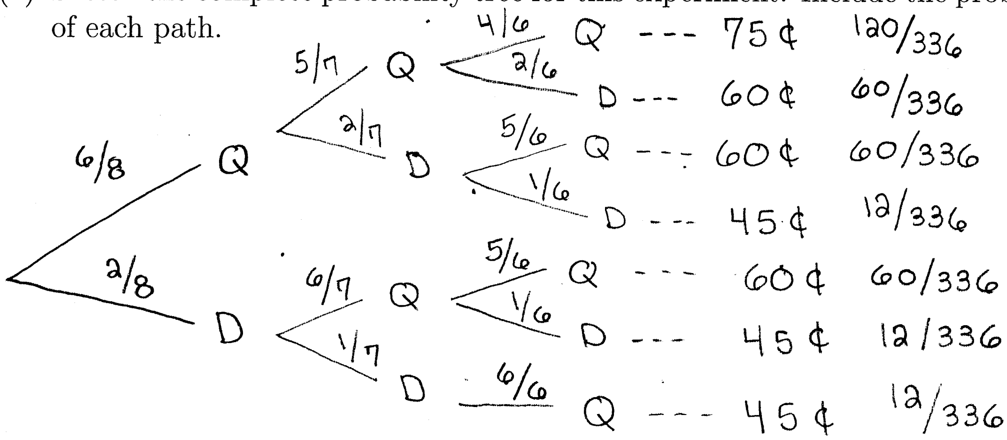
(c) Determine $P(B|\bar{B})$.

e CANNOT HAPPEN IF NOT e OCCURRED

$\boxed{0}$

9. (12 points) A jar contains 6 quarters and 2 dimes. Three coins are selected at random without replacement.

(a) Sketch the complete probability tree for this experiment. Include the probabilities of each path.



(b) Let the random variable x represent the value (in cents) of the three coin sample. What are the possible values of x?

45¢, 60¢, 75¢

(c) Refer to your tree diagram and determine the probability distribution of x. Write your probability distribution in the form of a table.

X	45	60	75
P(x)	$\frac{36}{336}$	$\frac{180}{336}$	$\frac{120}{336}$

(d) What is the expected value of the three coin sample?

$$45\left(\frac{36}{336}\right) + 60\left(\frac{180}{336}\right) + 75\left(\frac{120}{336}\right) = \frac{21420}{336} = \boxed{63.75¢}$$

10. (4 points) On average, a certain school's cafeteria has 418 customers per day. It is probably NOT a good idea to think that people visiting a cafeteria is a Poisson process. Briefly explain why not.

THE CUSTOMERS WILL SHOW UP MOSTLY AT TRADITIONAL MEAL TIMES, NOT UNIFORMLY THROUGHOUT THE DAY.

11. (10 points) In the U.S., yearly individual consumption of meat is approximately normally distributed with mean 218.4 lb and standard deviation 25.0 lb.

(a) What percent of Americans eat between 200 lb and 225 lb of meat in a year?

$P(200 < x < 225) = \text{normalcdf}(200, 225, 218.4, 25)$

$\approx 0.3732 = 37.32\%$

(b) What is the probability that a randomly selected American eats more than 250 lb of meat in a year?

$P(x > 250) = \text{normalcdf}(250, 999999, 218.4, 25)$

$\approx 0.1031 = 10.31\%$

(c) What is the probability that an American eats exactly 220 lb of meat in a year?

$P(x = 220) = \boxed{0}$

(d) What is an unusually small amount of meat consumed by an American in one year?

$\mu - 2\sigma = 218.4 - 50.0 = 168.4$

ANYTHING LESS THAN 168.4 lb

12. (8 points) The 24-hr customer service hotline of a local tech company receives, on average, 17.25 calls per day.

ASSUMING Poisson $\mu = \lambda = 17.25$

(a) On any given day, what is the probability that the hotline receives fewer than 10 calls?

$P(x<10) = P(x \le 9) = \text{poissoncdf}(17.25, 9)$

$\approx 0.0229 = 2.29\%$

(b) What is the probability that the hotline receives exactly 17 calls on a given day?

$P(x=17) = \text{poissonpdf}(17.25, 17) \approx 0.0961 = 9.61\%$

(c) Would it be unusual for the hotline to receive 25 calls in a day? Show work or explain.

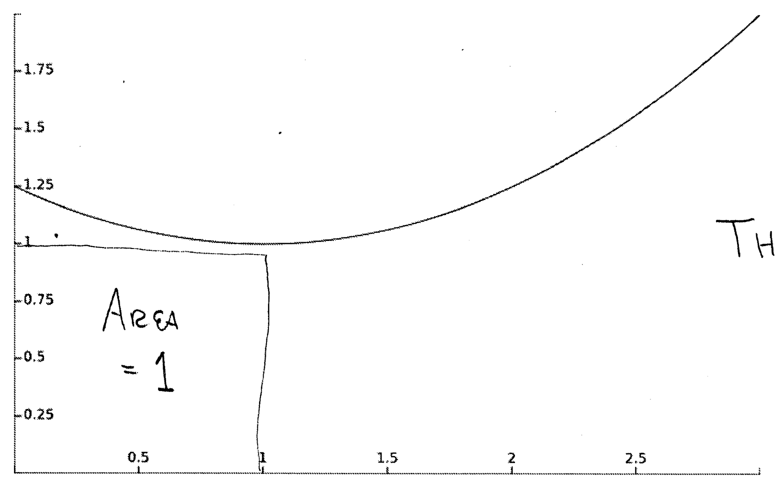
$\mu + 2\sigma = 17.25 + 2\sqrt{17.25}$

$\approx 25.56 \Rightarrow$ No, 25 IS NOT UNUSUAL, BUT 26 OR MORE ARE UNUSUAL.

13. (3 points) A person draws 5 marbles, with replacement, from a jar containing red marbles, green marbles, and blue marbles. Explain why this is probably **not** a binomial process.

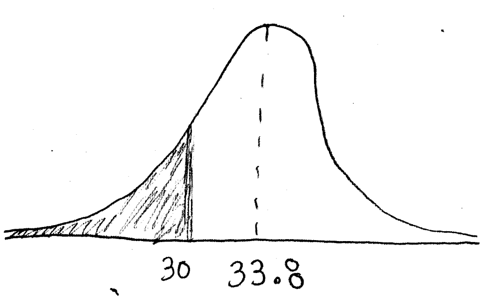
DEPENDING ON WHAT WE ARE OBSERVING, IT SEEMS THAT THERE ARE MORE THAN 2 OUTCOMES PER TRIAL (RED, GREEN, BLUE).

14. (3 points) Explain why the graph shown below cannot be a probability density curve.



THE AREA UNDER THE CURVE EXCEEDS 1.

15. (6 points) Tarsus lengths of adult male grackles are normally distributed with mean 33.80 mm and standard deviation 4.84 mm. Roughly sketch the probability density curve associated with this distribution. Shade the area that corresponds to the probability $P(x < 30)$. Then compute the probability.



$P(x<30) =$

$\text{normalcdf}(-\infty, 30, 33.8, 4.84)$

$\approx \text{normalcdf}(-999999, 30, 33.8, 4.84)$

$= 0.2162$

$= 21.62\%$