Math 153 - Final Exam

December 11, 2018

Name key

Score ________

Show all work to receive full credit. Supply explanations where necessary.

1. (16 points) Media researchers report that the amounts of time that U.S. adult males spend watching television each day are normally distributed with mean 4.28 hours and standard deviation 1.30 hours.

   (a) If a U.S. adult male is selected at random, what is the probability that his daily TV viewing exceeds 5 hours?

   $P(x>5) = \text{normalcdf}(5, 999999, 4.28, 1.30) \approx \boxed{0.2898}$

   (b) What is the probability that a random U.S. adult male spends exactly 4 hours watching TV on a given day?

   $P(x=4) = \boxed{0}$

   (c) If 200 U.S. adult males are selected at random, about how many spend between 3 and 4 hours watching television each day?

   $200 * P(3<x<4) = 200 * \text{normalcdf}(3, 4, 4.28, 1.30) \approx 50.465 \rightarrow \boxed{\text{About } 50}$

   (d) How much TV would a U.S. adult male have to watch per day in order to be at the 99th percentile?

   $\text{invNorm}(0.99, 4.28, 1.30) \approx \boxed{7.30 \text{ Hours}}$

2. (8 points) Shown below are the prices (in dollars) listed for the TI-84 Plus graphing calculator at a variety of online shopping sites.

   104.95, 111.99, 114.97, 124.99, 139.76, 199.95

   Compute the sample mean and standard deviation. Then use them to determine unusually small and large prices for the calculator.

TI-83...

$\bar{X} \approx 132.77$

$S \approx 35.06$

$\bar{X} - 2s \approx 62.65$

$\bar{X} + 2s \approx 202.89$

Unusually small prices are less than \$62.65,

Large prices are greater than \$202.89

3. (9 points) For each situation, decide whether the distribution of **sample means** is approximately normal. If so, find the mean and standard deviation of the sampling distribution.

   (a) The cholesterol content of eggs is approximately normally distributed with mean 215 mg and standard deviation of 15 mg. Samples of 15 eggs are randomly selected.

   Yes, because population is normal. CLT #2

   $\mu_{\bar{x}} = 215$

   $\sigma_{\bar{x}} = 15/\sqrt{15}$

   (b) Adult male wombats in Narawntapu National Park have a mean weight of 35.6 kg with a standard deviation of 2.8 kg. Random samples of 45 male wombats are obtained.

   Yes, because sample size is large enough

   $n = 45 > 30$ CLT #1

   $\mu_{\bar{x}} = 35.6$

   $\sigma_{\bar{x}} = 2.8/\sqrt{45}$

   (c) Speeds on a highway have a mean of 70.3 mph and a standard deviation of 3.2 mph. Samples of 20 cars are picked at random.

   No CLT does not apply.

4. (12 points) According to data collected by the National Athletic Trainers' Association, a high school football quarterback receives an average of 0.51 injuries per season. Assume the numbers of injuries are Poisson distributed.

   (a) In any given season, what is the probability that a quarterback is not injured at all?

   $$P(x=0) = \text{poissonpdf}(0.51, 0) = \boxed{0.6005}$$

   (b) What is the probability that a quarterback is injured 2 or more times in a season?

   $$P(x \geq 2) = 1 - P(x \leq 1) = 1 - \text{poissoncdf}(0.51, 1) = \boxed{0.0933}$$

   (c) In any given season, what would be an unusually large number of injuries for a quarterback?

   $$\mu + 2\sigma = 0.51 + 2\sqrt{0.51} \approx 1.94$$

   $\Rightarrow$ 2 or more injuries are unusual.

5. (12 points) The following table summarizes the numbers of years lived by past U.S. presidents after their first inauguration.

Years Lived	Frequency
0–4	8
5–9	2
10–14	5
15–19	7
20–24	4
25–29	6
30–34	0
35–39	1

33

(a) What is the class width?

5

(b) Determine the class midpoints.

2, 7, 12, 17, 22, 27, 32, 37

(c) Use class midpoints to compute the (weighted) mean.

$$\bar{X} \approx \frac{2(8) + 7(2) + 12(5) + 17(7) + 22(4) + 27(6) + 32(0) + 37(1)}{33}$$

$$= \boxed{15.\overline{03}}$$

(d) Use class midpoints to compute the (weighted) median.

MEDIAN $\approx$ 17^{TH} MIDPOINT $= \boxed{17}$

6. (8 points) Boxing heavyweights Deontay Wilder and Tyson Fury will fight a rematch on March 30, 2019. The current odds in favor of Wilder are 6:5. What are the odds against Wilder? Based on the odds, what is the probability that Wilder wins?

ODDS IN FAVOR ARE $\frac{6}{5}$ $\Longrightarrow$

ODDS AGAINST ARE $\frac{5}{6}$

PROB OF WINNING $= \frac{6}{11}$.

7. (4 points) Decide whether the probability is theoretical (classical) or experimental (empirical).

   (a) While at the casino, Brittany won 10 hands of blackjack after playing 15 games. She claims that her probability of winning is 10/15.

   EXPERIMENTAL

   (b) In a single birth, the probability of having a girl is 50%.

   THEORETICAL

8. (16 points) According to research conducted in late 2018, 60% of Massachusetts voters believe that Sen. Elizabeth Warren's claim about Native American ancestry is "not at all important." Thirty (30) Massachusetts voters are randomly selected.

BINOMIAL

$p = 0.60$
$q = 0.40$
$n = 30$

   (a) Even if the sampling is done without replacement, it is reasonable to assume that trials are independent. Why?

   30 VOTERS IS FAR LESS THAN 5% OF THE POPULATION (5% RULE!)

   (b) What is the probability that exactly 30 voters in the sample believe Warren's claim is not important?

   $$P(x=30) = \text{binompdf}(30, 0.60, 30) = 2.21 \times 10^{-7} \approx \boxed{0}$$

   (c) What is the probability that fewer than 20 voters in the sample believe Warren's claim is not important?

   $$P(x<20) = \text{binomcdf}(30, 0.60, 19) = \boxed{0.7085}$$

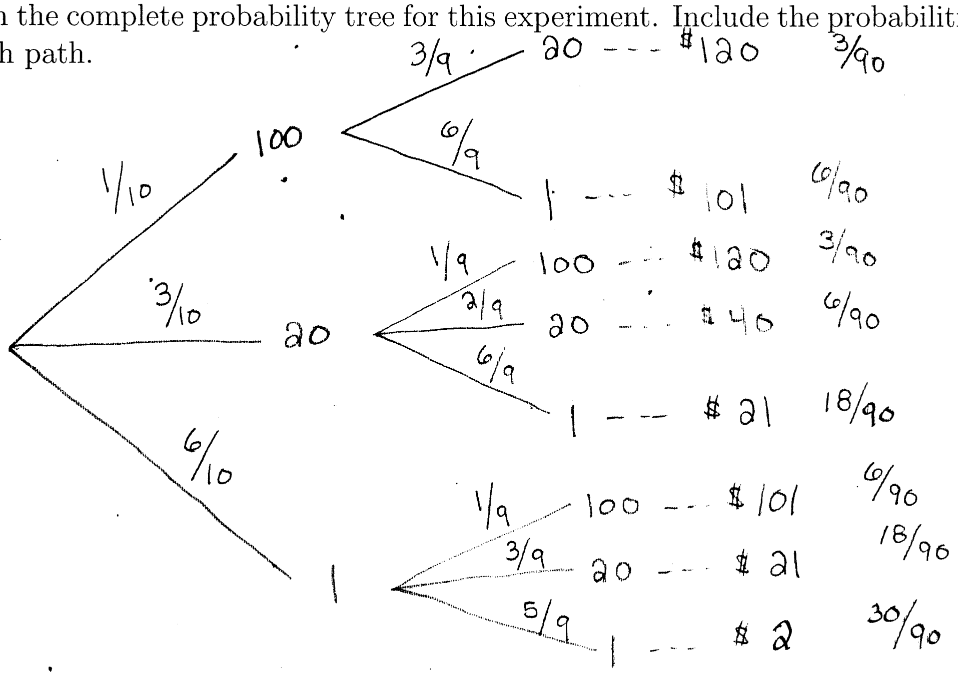
   (d) What is the CV in samples of size 30?

   $$\mu = np = 18$$
   $$\sigma = \sqrt{npq} = 2.68$$
   $$CV = \frac{\sigma}{\mu} = \frac{\sqrt{npq}}{np} = \frac{\sqrt{q}}{\sqrt{np}} \approx \boxed{14.9\%}$$

10 BILLS

9. (15 points) An envelope contains one (1) \$100 bill, three (3) \$20 bills, and six (6) \$1 bills. Two bills are selected at random without replacement.

(a) Sketch the complete probability tree for this experiment. Include the probabilities of each path.



(b) Let the random variable x represent the value (in dollars) of the two bill sample. What are the possible values of x?

2, 21, 40, 101, 120

(c) Is x a discrete or continuous random variable?

(d) Refer to your tree diagram and determine the probability distribution of x. Write your probability distribution in the form of a table.

X	2	21	40	101	120
P(x)	$\frac{30}{90}$	$\frac{36}{90}$	$\frac{6}{90}$	$\frac{12}{90}$	$\frac{6}{90}$

(e) What is the expected value of the two bill sample?

$$\mu = 2\left(\frac{30}{90}\right) + 21\left(\frac{36}{90}\right) + 40\left(\frac{6}{90}\right) + 101\left(\frac{12}{90}\right) + 120\left(\frac{6}{90}\right)$$

$$= \frac{2988}{90} = \boxed{\$33.2}$$

10. (12 points) The mean number of daily email messages received by each InTech employee is 63.4, with a standard deviation of 11.9. A group of 32 employees set up their own email server for work on a special project.

(a) What is the probability that the mean number of email messages for the special project group exceeds 66?

$$P(\bar{x} > 66) = \text{normalcdf}(66, 999999, 63.4, 11.9/\sqrt{32})$$

$$= 0.1082$$

(b) What would be an unusually large mean number of email messages for the group?

$$\mu_{\bar{x}} + 2\sigma_{\bar{x}} = 63.4 + 2\,11.9/\sqrt{32}$$

$$= 67.6$$

ANY MEAN GREATER THAN 67.6 IS UNUSUAL.

(c) Did you use the Central Limit Theorem for problems (a) and (b)? If so, how and why?

YES, USED CLT #1 $(n > 30)$.

$$\mu_{\bar{x}} = \mu = 63.4$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = 11.9/\sqrt{32}$$

11. (6 points) After nine weeks of work, Sam's weekly earnings have a mean of \$115.87. How much will Sam's mean increase if his tenth-week paycheck is for \$157.25?

$$\frac{115.87(9) + 157.25}{10} = \frac{1200.08}{10} = 120.01$$

$$120.01 - 115.87 = \$4.14$$

12. (10 points) In a random sample of 500 American fathers, it was found that 40.4% of the fathers were also grandfathers.

TI-83

(a) Compute a 90% confidence interval estimate for the true proportion of American fathers who are also grandfathers. State your conclusion in a complete sentence.

1-PropZInt

$X = 40.4\%$ OF $500 = 202$

$n = 500$

C-Level $= 0.90$

THE 90% C.I. ESTIMATE IS

$(0.3679, 0.4401)$

WE ARE 90% CONFIDENT THAT THE PROPORTION OF FATHERS WHO ARE GRANDFATHERS IS BETWEEN 36.79% AND 44.01%.

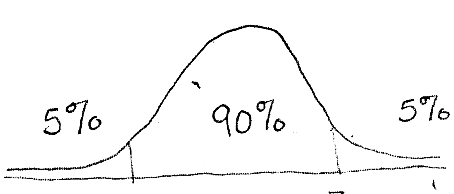
(b) What is the margin of error in your confidence interval estimate?

$$E = \frac{0.4401 - 0.3679}{2} = \boxed{0.0361 = 3.61\%}$$

13. (8 points) As a manager for an advertising company, you must plan a campaign designed to increase Twitter usage. You want to first determine the percentage of adults who are familiar with Twitter. How many adults must you survey in order to be 90% confident that your estimate is within five percentage points of the true population percentage? (Assume that nothing is known about the percentage of adults familiar with Twitter.)

$$n = \frac{z_{\alpha/2}^2 \times 0.25}{E^2} = \frac{(1.645)^2 (0.25)}{(0.05)^2} = 270.6025$$

$\boxed{\text{Use } n = 271}$



$z_{\alpha/2} = \text{invNorm}(0.95) = 1.645$

14. (14 points) The mean age of passenger cars in the United States is 8.4 years. A random sample of 58 cars parked in the lot of a very large manufacturing plant gave a sample mean and sample standard deviation of 9.5 years and 3.1 years, respectively. We would like to conclude that the mean age of cars driven by the plant's employees is greater than the national average.

(a) State the null and alternative hypotheses.

CLAIM: $\mu > 8.4$

COUNTER: $\mu \leq 8.4$

$\Rightarrow$

$H_0: \mu = 8.4$

$H_1: \mu > 8.4$

(b) Compute the test statistic.

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{9.5 - 8.4}{3.1/\sqrt{58}} \approx 2.702$$

(c) Carry out the test of the the claim at the level $\alpha = 0.05$. (If you intend to compute critical values, you may approximate them using invNorm rather than invT.)

TI-83

T-Test

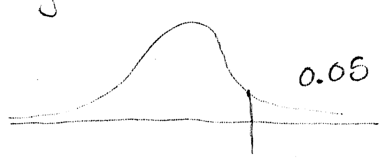
P-VALUE = $0.0045 < \alpha$

$\Downarrow$

Reject Ho.

THE EVIDENCE DOES NOT SUPPORT $\mu = 8.4$.

RIGHT-TAILED T-TEST



$= 1.672...$

invT(0.95, 57)

$\approx$ invNorm(0.95)

≈ 1.645

$t > 1.645$

Reject Ho.