Math 153 - Test 3a
April 25, 2013

Name key Score

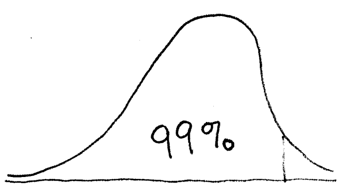
Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) A *USA Today* survey found that 83% of government employees use email. In a government agency department of 40 people, what is the probability that fewer than 35 people use email?

Binomial
$p = 0.83$
$N = 40$

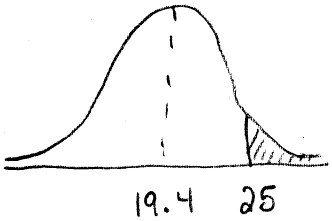
$$P(x < 35) = P(x \le 34)$$
$$= \text{binomialcdf}(40, 0.83, 34)$$
$$\approx \boxed{0.6958 = 69.58\%}$$

2. (4 points) Lifetimes of wristwatches are normally distributed with mean 25 months and standard deviation 5 months. How long should a wristwatch last if it is to outlive 99% of all wristwatches?



$$X = \text{invNorm}(0.99, 25, 5) \approx \boxed{36.63 \text{ months}}$$

3. (4 points) The ages of Amtrak passenger train cars are normally distributed with mean 19.4 years and standard deviation 4.05 years. In a certain state, there are approximately 450 Amtrak train cars in use. About how many are older than 25 years?



$$450 \times P(x > 25)$$

$$= 450 \times \text{normalcdf}(25, 99999, 19.4, 4.05)$$

$$\approx 37.5 \approx \boxed{38}$$

4. (4 points) In the United States from 1850 to 2000, there were an average of 17.93 hurricanes per decade. In any given decade, what is the probability that there are 19 or more hurricanes?

Poisson

$\mu = 17.93$

$$P(x \geq 19) = 1 - P(x \leq 18)$$

$$= 1 - \text{poissoncdf}(17.93, 18)$$

$$\approx 0.4312 = 43.12\%$$

5. (3 points) A 98% confidence interval estimate for the proportion of business majors at a certain university is (0.188, 0.287). Find the best estimate for the true population proportion and the margin of error.

$$\hat{p} = \frac{0.188 + 0.287}{2} = 0.2375$$

$$E = 0.2375 - 0.188 = 0.0495$$

6. (2 points) What is the "magic number" for the sample size in the Central Limit Theorem that describes when a sampling distribution will be approximately normal?

$\boxed{30}$

7. (4 points) A survey of 50 first-time white-water canoers showed that 23 did not want to repeat the experience.

   (a) Find a 90% confidence interval estimate for the true population proportion of canoers who did not wish to canoe the rapids a second time.

1-PropZInt

X = 23

N = 50

90%

C.I. is (0.34406, 0.57594)

   (b) A rafting company has approximately 872 customers per year. If the company prints special brochures to market to repeat customers, what is the minimum number of brochures it should print each year?

$0.34406 \times 872 \approx \boxed{300}$

Math 153 - Test 3b
April 25, 2013

Name key

Score

Show all work to receive full credit. Supply explanations where necessary. This portion of the test is due Tuesday, April 30. You must work individually on this test.

1. (9 points) A *USA Today* survey found that 21% of Americans watch fireworks on July 4. A random sample of 200 Americans is selected.

   (a) What is the probability that at least 50 Americans in the sample watch fireworks?

   BINOMIAL

   $p = 0.21$

   $N = 200$

   $P(x \geq 50) = 1 - P(x < 50) = 1 - P(x \leq 49)$

   $= 1 - \text{binomialcdf}(200, 0.21, 49)$

   $= 0.09821 \approx 9.8\%$

   (b) In samples of size 200, what is the mean number of Americans who watch fireworks? What is the standard deviation?

   $\mu = 200 \times 0.21$

   $= 42$

   $\sigma = \sqrt{200 \times 0.21 \times 0.79}$

   $= \sqrt{33.18} \approx 5.76$

   (c) In a sample of size 200, what are unusually small and large numbers of Americans who watch fireworks?

   $\mu - 2\sigma = 30.48$

   $\mu + 2\sigma = 53.52$

   UNUSUALLY SMALL $\Leftrightarrow$ LESS THAN 30.48

   UNUSUALLY LARGE $\Leftrightarrow$ MORE THAN 53.52

2. (7 points) The numbers of daily passengers on an express bus are normally distributed with mean 48 and standard deviation 3. On any given day, what is the probability that an express bus has more than 55 passengers or fewer than 43 passengers?

   normalcdf(55, 99999, 48, 3) + normalcdf(-99999, 43, 48, 3)

   $= 0.0576 \approx 5.8\%$

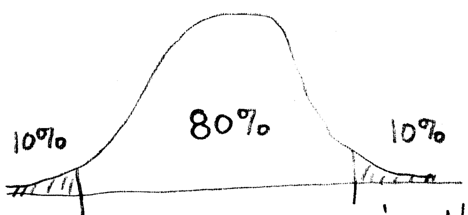
3. (4 points) Nearly one-half of Americans aged 25 to 29 are unmarried. How large a sample is necessary to estimate the true proportion of unmarried Americans in this age group within $2\frac{1}{2}$ percentage points with 90% confidence?

90% C.I. $\Rightarrow$ $z_{\alpha/2} = 1.645$

$$N = \frac{(z_{\alpha/2})^2 (0.5)(0.5)}{(0.025)^2} = \frac{(1.645)^2 (0.25)}{(0.025)^2} = 1082.41$$

$\boxed{\text{CHOOSE } N = 1083}$

4. (8 points) A contractor decided to build homes that will include the middle 80% of the market. If the average size of homes built is 1810 square feet, find the minimum and maximum sizes of the homes the contractor should build. Assume home sizes are normally distributed with standard deviation 92 square feet.



invNorm (0.10, 1810, 92) $= \boxed{1692.1 \text{ FT}^2 \leftarrow \text{MIN}}$

invNorm (0.90, 1810, 92) $= \boxed{1927.9 \text{ FT}^2 \leftarrow \text{MAX}}$

5. (7 points) It has been reported that about 11% of U.S. elementary students attend private school. A random sample of 450 students indicated that 55 attended private school. Use a 95% confidence interval to estimate the true population proportion of students attending private school. What can you say about the accuracy of the reported percentage?

1-PropZInt

X = 55

N = 450

C-LEVEL = 0.95

$\boxed{\text{C.I. IS } (0.09196, 0.15249)}$

SINCE 11% IS INSIDE THE C.I., THE REPORTED PERCENTAGE CANNOT BE REJECTED.

6. (9 points) At a certain hospital, there are an average of 210 low-birth-weight babies born each year.

   (a) Find the mean number of low-birth-weight babies born per day.

$$\mu = \frac{210}{365} \approx 0.5753$$

   (b) On any given day, what is the probability that more than one low-birth-weight baby is born?

POISSON

$\mu = 0.5753$

$$P(x > 1) = 1 - P(x \leq 1)$$
$$= 1 - \text{poissoncdf}(0.5753, 1)$$
$$\approx \boxed{11.4\%}$$

   (c) Is it unusual to have more than one low-birth-weight baby born at the hospital on any given day?

NO, THE PROBABILITY IS ≈ 11%. NOT UNUSUAL!

THE CUTOFF FOR UNUSUAL VALUES IS 5%.

7. (7 points) Yearly individual consumption of meat is normally distributed with mean 218.4 lb and standard deviation 25 lb.

   (a) In a sample of 40 individuals, find the probability that the mean meat consumption is greater than 224 lb.

$$\text{normalcdf}(224, 99999, 218.4, 25/\sqrt{40})$$
$$= 0.078285 \approx 7.8\%$$

   (b) What is the probability if the sample size is changed to 400?

$$\text{normalcdf}(224, 99999, 218.4, 25/\sqrt{400})$$
$$= 0.0000037355$$

IN A SAMPLE OF 400, THE PROBABILITY IS NEARLY 0.

8. (10 points) One number is selected from each box.

1	5	6

4	8

(a) List all possible samples and find the range of each sample.

$\{1,4\} \to 3$

$\{1,8\} \to 7$

$\{5,4\} \to 1$

$\{5,8\} \to 3$

$\{6,4\} \to 2$

$\{6,8\} \to 2$

(b) Summarize the sampling distribution of ranges in a probability distribution table.

X	P(x)
1	1/6
2	2/6
3	2/6
7	1/6

(c) Find the mean of the sample ranges.

$$1\left(\frac{1}{6}\right) + 2\left(\frac{2}{6}\right) + 3\left(\frac{2}{6}\right) + 7\left(\frac{1}{6}\right)$$

$$= \frac{18}{6} = \boxed{3}$$

(d) Find the population range.

$$8-1 = \boxed{7}$$

(e) Do the sample ranges target the population range? Explain.

NO, THE POPULATION RANGE IS 7 AND THE SAMPLE RANGES HAVE MEAN 3.

9. (6 points) In a random sample of 35 U.S. cities, the mean growing season was 190.7 days. Assuming the population standard deviation is 54.2 days, find a 95% confidence interval estimate for the true mean growing season.

ZInterval

$\sigma = 54.2$

$\bar{X} = 190.7$

$N = 35$

C-LEVEL: 0.95

$\boxed{\text{C.I. is } (172.74, 208.66)}$

10. (8 points) Suppose you wish to estimate the mean number of chocolate chips per cookie for a large national brand. How many cookies would have to be sampled to estimate the true mean number of chips per cookie within 2 chips with 98% confidence? Assume that $\sigma = 10.1$ chips.

$$N = \left(\frac{Z_{\alpha/2}\,\sigma}{E}\right)^2$$

$\alpha = 0.02 \Rightarrow \alpha/2 = 0.01$

$\Rightarrow Z_{\alpha/2} = \text{invNorm}(0.99)$

$= 2.326$

$$\left[\frac{(2.326)(10.1)}{2}\right]^2 = 137.97556$$

$$\approx \boxed{138}$$