

Math 153 - Test 2

March 9, 2017

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary.

1. (6 points) Fred took the SAT, and his math score was 676 on a test with mean 500 and standard deviation 110. Judy took the ACT, and her math score was 31 on a test with mean 20 and standard deviation 6. Compute the z-scores for both Fred and Judy. Who scored better?

Fred:

$$z = \frac{676 - 500}{110} = 1.60$$

Judy:

$$z = \frac{31 - 20}{6} \approx 1.83$$

Judy scored a bit better.

2. (4 points) The coefficient of variation in gold prices through the year 1993 was 5.90%. In the same time period, the mean gold price was \$364.42 per ounce. Find the standard deviation in gold prices.

$$\frac{S}{364.42} = 5.90\% \Rightarrow S \approx \$21.50$$

3. (8 points) A random sample of 18 new Aldi Ultracell batteries was obtained. The batteries were drained in two different tests. Nine batteries were pulse tested and nine batteries were time tested. The data are shown below.

Pulses: 638, 645, 636, 651, 639, 649, 654, 627, 644

Time (hr): 8.76, 8.81, 8.81, 8.7, 8.73, 8.76, 8.68, 8.64, 8.79

Compute the coefficient of variation (CV) for each set of data. Which has more spread, life in pulses or life in time?

Pulses:

$$\bar{X} \approx 642.56$$

$$S \approx 8.44$$

$$CV \approx \frac{8.44}{642.56} \approx 1.31\%$$

More spread in pulses.

Time:

$$\bar{X} \approx 8.74$$

$$S \approx 0.0595$$

$$CV \approx \frac{0.0595}{8.74} \approx 0.681\%$$

Less spread in life in time.

4. (9 points) The selling prices of new homes in the Saratoga Falls subdivision are given (in numerical order) on the attached sheet.

(a) At what percentile is a home that sells for \$272 thousand?

$$\frac{31}{40} = 0.775 \Rightarrow \text{AT ABOUT THE 78}^{\text{TH}} \text{ PERCENTILE.}$$

(b) Find the value at the 35th percentile.

$$\frac{L}{40} = 0.35 \Rightarrow L = 14$$

$$\frac{14^{\text{TH}} + 15^{\text{TH}}}{2} = \frac{209 + 219}{2} = 214$$

\$ 214 THOUSAND

(c) Find the value at the 83rd percentile.

$$\frac{L}{40} = 0.83 \Rightarrow L = 33.2$$

$$34^{\text{TH}} = \$ 275 \text{ THOUSAND}$$

5. (10 points) The following table shows the numbers of chocolate chips counted in each of 24 Hannaford Chocolate Chip cookies.

DATA IN NUMERICAL ORDER

11 12 12 12 13 13 13 14 14 14 14
 14 14 15 15 15 15 16 16 16 16 17 21*

Determine the quartiles, the interquartile range, and the cutoff values for outliers. Then sketch the modified boxplot.

$$Med = \frac{14 + 14}{2} = 14$$

$$IQR = 15.5 - 13 = 2.5$$

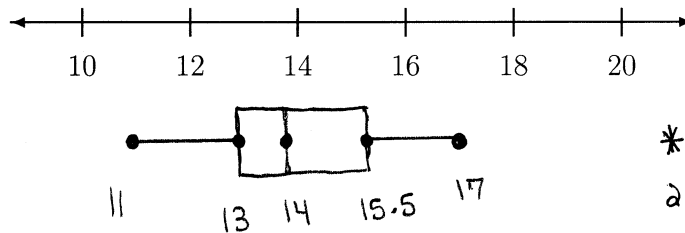
$$Q_1 = \frac{13 + 13}{2} = 13$$

$$\text{CUTOFFS: } 13 - 1.5(2.5) = 9.25$$

$$15.5 + 1.5(2.5) = 19.25$$

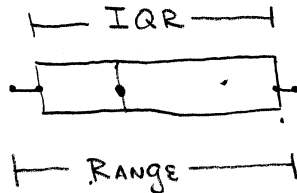
$$Q_3 = \frac{15 + 16}{2} = 15.5$$

21 IS AN OUTLIER.

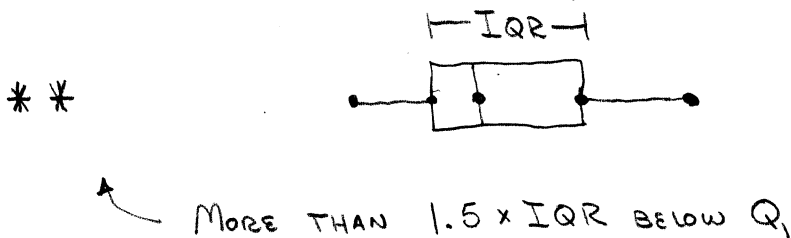


6. (6 points) Think carefully about the about the characteristics of a modified boxplot. For each part of this problem, sketch a boxplot that would correspond to a data set with the given properties.

(a) The range is just a little bit greater than the IQR.



(b) There are two outliers in the lower extreme.



7. (4 points) A PSC student is selected at random. Let A be the event that the student is taking a math class. Let B be the event that the student is a female. Are A and B disjoint (mutually exclusive)? Explain.

→ No, THERE ARE FEMALES IN MATH 153

SO THERE MUST BE FEMALE MATH STUDENTS.

8. (4 points) Suppose the wicked witch of the east is hanging out at a random location in her big yard which measures 120 ft by 310 ft. Dorothy Gale's little house measures 12 ft by 10 ft, and a tornado is about to hurl the house into the witch's yard. What is the probability that the witch is smashed by the falling house? What type of probability are you assigning?

$$120 \text{ FT} \times 310 \text{ FT} = 37200 \text{ FT}^2$$

$$12 \text{ FT} \times 10 \text{ FT} = 120 \text{ FT}^2$$

$$\text{GEOMETRIC PROB IS } \frac{120}{37200} \approx 0.323\%$$

$$P(A \cap B) = 0$$

9. (3 points) Indicate whether each statement is true or false.

(a) FALSE If A and B are disjoint (mutually exclusive), then $P(A \cup B) = 0$.

(b) True If D is an impossible event, then \bar{D} is a certain event.

(c) FALSE If $P(X) = 0.4$ and $P(Y) = 0.3$, then it must be true that $P(X \cup Y) = 0.7$.
Only if $X \cap Y = \phi$

10. (8 points) Suppose A and B are events such that $P(A) = 0.46$, $P(B) = 0.68$, and $P(A \cup B) = 0.92$. Find each of the following.

(a) $P(A \cap B)$
 $= P(A) + P(B) - P(A \cup B) = 0.22$

(b) $P(\bar{A}) = 1 - 0.46 = 0.54$

(c) $P(A \cup \bar{A}) = 1$

(d) $P(\overline{A \cup B}) = 1 - 0.92 = 0.08$

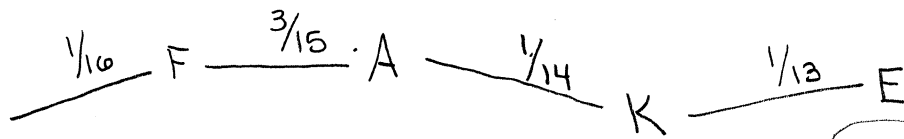
11. (4 points) Given the probabilities $P(A) = 0.45$ and $P(B) = 0.65$, determine two things wrong with the conclusion $P(A \cup B) = 0.45 + 0.65$.

① $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ INTERSECTION WAS NEGLECTED

② $0.45 + 0.65 = 1.10$

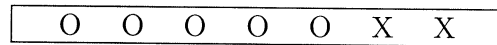
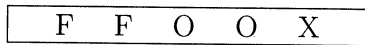
4 AND A PROB CANNOT EXCEED 1.

12. (5 points) Four letters are selected at random without replacement from the word EYJAFJALLAJOKULL. What is the probability of spelling the word FAKE?

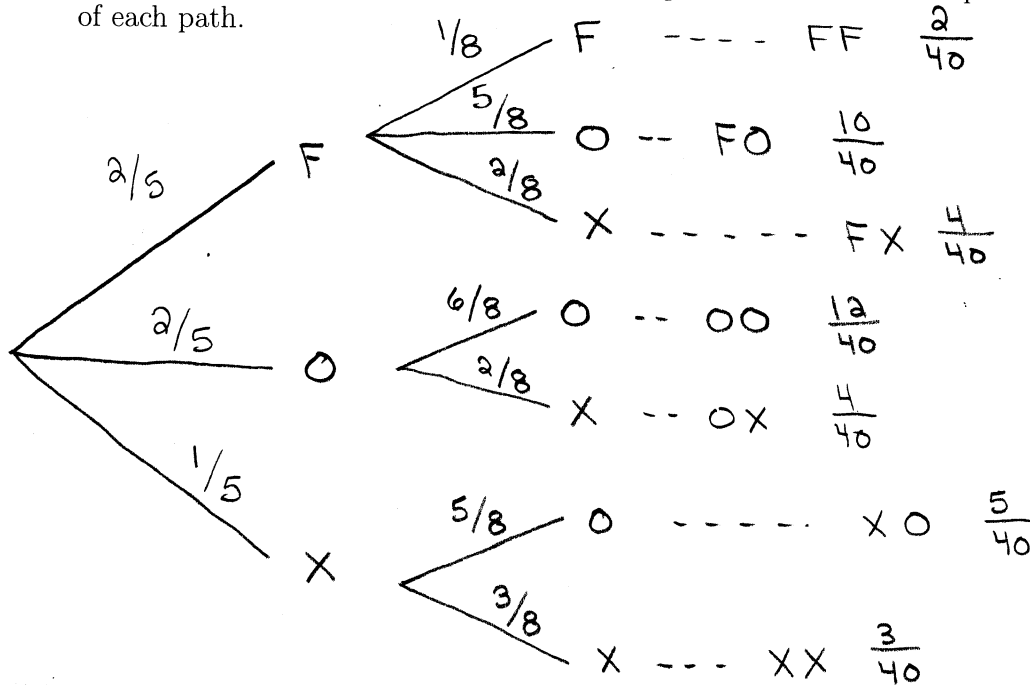


$$\frac{1}{16} \times \frac{3}{15} \times \frac{1}{14} \times \frac{1}{13} = \frac{3}{43680}$$

13. (12 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box.



- (a) Sketch the complete tree diagram for this experiment. Include the probabilities of each path.



- (b) What is the probability of selecting at least one letter O?

$$\{FO, OO, OX, XO\}$$

$$\frac{10 + 12 + 4 + 5}{40} = \frac{31}{40}$$

- (c) What is the probability of selecting at least one letter that is not F?

All BUT FF \Rightarrow

$$1 - \frac{2}{40} = \frac{38}{40}$$

14. (10 points) During the last flu season, a number of adults participated in a double-blind study of the effectiveness of a new flu vaccine. The following data were collected.

	Caught the flu	Did not catch the flu	
Took vaccine	37	259	296
Took placebo	97	232	329
	134	491	625

A person from this study is selected at random.

- (a) What is the probability that the person caught the flu?

$$\frac{134}{625} = 21.44\%$$

- (b) What is the probability that the person took the placebo?

$$\frac{329}{625} = 52.64\%$$

- (c) What is the probability that the person took the vaccine and caught the flu?

$$\frac{37}{625} = 5.92\%$$

- (d) What is the probability that the person did not take the placebo?

$$\frac{296}{625} = 47.36\%$$

- (e) What is the probability that the person took the vaccine or caught the flu?

$$\frac{37 + 259 + 97}{625} = \frac{393}{625} = 62.88\%$$

15. (7 points) A letter is selected at random from the first box and placed into the second box. Then a letter is selected at random from the second box. The outcomes are recorded as ordered pairs of letters such as (a, b) . (This problem is about **events** not probabilities.)

a a a b b c

a a b b b b

- (a) Find the sample space for this experiment.

$$\{(a, a), (a, b), (b, a), (b, b), (c, a), (c, b), (c, c)\}$$

- (b) Is each outcome in your sample space equally likely? Explain.

NO, IN FACT (c, c) IS LESS LIKELY THAN ALL OTHER OUTCOMES BECAUSE OF ONLY 1 c IN BOX 1.

- (c) Let X be the event of drawing the letter b from the first box. What is \bar{X} ?

$$\begin{aligned} \bar{X} &= \text{EVENT OF DRAWING } a \text{ OR } c \text{ FROM 1ST BOX} \\ &= \{(a, a), (a, b), (c, a), (c, b), (c, c)\} \end{aligned}$$

- (d) Let Y be the event of drawing the letter c from the second box. What is $X \cup Y$?

$$\begin{aligned} X \cup Y &= \text{EVENT OF DRAWING } b \text{ FROM 1ST OR } c \text{ FROM 2ND} \\ &= \{(b, a), (b, b), (c, c)\} \end{aligned}$$

- (e) What is $X \cap Y$?

↳ b FROM 1ST AND c FROM 2ND IS IMPOSSIBLE

$$X \cap Y = \phi$$

Housing Prices in Saratoga Falls

(In thousands of dollars)

143	145	186	192	192	192	194	195	199	201
202	202	205	209	219	221	221	221	230	232
233	235	238	240	245	246	249	257	268	270
271	272	272	275	280	287	324	345	346	387