Math 153 - Test 3

April 20, 2017

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (15 points) The probability distribution for the random variable x is shown below.

x	0	1	2	3	4	5
$P(x)$	0.04	0.09	0.31	0.48	0.02	0.06

(a) What two things about the table above show that it is a probability distribution?

(1) $0 \le P(x) \le 1$

(2) $\sum P(x) = 1$

(b) What is the mean value of x?

$$\mu = 0(0.04) + 1(0.09) + 2(0.31) + 3(0.48) + 4(0.02) + 5(0.06)$$

$$= \boxed{2.53}$$

(c) What is the standard deviation in x?

$$\sigma^2 = 0(0.04) + 1(0.09) + 4(0.31) + 9(0.48) + 16(0.02) + 25(0.06) - (2.53)^2$$

$$\sigma^2 = 1.0691$$

$$\boxed{\sigma \approx 1.034}$$

(d) Use the mean and standard deviation to determine the unusual values of x.

$$\mu - 2\sigma \approx 0.462$$

$$\mu + 2\sigma \approx 4.598$$

$\Rightarrow$ 0 & 5 ARE UNUSUAL

(e) Use the 5% rule to determine the unusual values of x.

Using 5% RULE, ONLY 0 IS UNUSUAL.

2. (9 points) In the United States, 1 in 6 people have light blue eyes. 90 Americans are randomly selected.

BINOMIAL

(a) What is the probability that 18 have light blue eyes?

$$P(x=18) = \text{binompdf}(90, 1/6, 18)$$

$$\approx 0.0743 = \boxed{7.43\%}$$

(b) What is the probability that at least 20 have light blue eyes?

$$P(x \geq 20) = 1 - P(x < 20) = 1 - P(x \leq 19)$$

$$= 1 - \text{binomcdf}(90, 1/6, 19) \approx 0.1043 = \boxed{10.43\%}$$

(c) In the sample of 90, what would be an usually large number of people with light blue eyes? (Be sure to show your work.)

$$\mu + 2\sigma = np + 2\sqrt{npq}$$

$$= 90\left(\tfrac{1}{6}\right) + 2\sqrt{90\left(\tfrac{1}{6}\right)\left(\tfrac{5}{6}\right)}$$

$$\approx 22.07 \implies \boxed{23 \text{ OR MORE}}$$

3. (3 points) Given the following discrete probability distribution, determine the value of $P(2 \leq x < 5)$.

x	0	1	2	3	4	5	6
$P(x)$	0.03	0.02	0.18	0.35	0.38	0.03	0.01

$$P(2 \leq x < 5) = P(2) + P(3) + P(4)$$

$$= 0.18 + 0.35 + 0.38 = \boxed{0.91}$$

4. (3 points) A person draws 6 marbles, without replacement, from a jar containing a small number of red marbles and blue marbles. Explain why this is definitely **not** a binomial process.

THE TRIALS ARE NOT INDEPENDENT.

SINCE THE EXPERIMENT IS DONE WITHOUT REPLACEMENT, THE PROBABILITIES CHANGE WITH EACH SELECTION.

5. (3 points) Approximately 10% of the world's population are left-handed. However, there is a general tendency that the more violent a particular society is, the higher the proportion of left-handers. For example, there is a very high homicide rate among the Eipo of Indonesia, and 27% of the Eipo are left-handed. In a group of 35 Eipo, what is the probability that fewer than 8 are left-handed?

BINOMIAL

$$P(x<8) = P(x \le 7)$$
$$= \text{binomcdf}(35, 0.27, 7)$$
$$\approx 0.2333 = 23.33\%$$

6. (12 points) In the state of Illinois, there are about 79 traffic fatalities per month.

(a) What is the probability that there are exactly 79 traffic fatalities in any given month?

Poisson $\mu = 79$

$$P(x=79) = \text{poissonpdf}(79, 79)$$
$$\approx 0.0448 = 4.48\%$$

(b) In any given month, what is the probability that there are more than 88 traffic fatalities?

$$P(x>88) = 1 - P(x \le 88)$$
$$= 1 - \text{poissoncdf}(79, 88) \approx 0.1430$$
$$= 14.30\%$$

(c) In July 2011, there were 101 traffic fatalities. Is this an unusually large number of fatalities? Explain.

$$\mu + 2\sigma = 79 + 2\sqrt{79} \approx 96.78$$

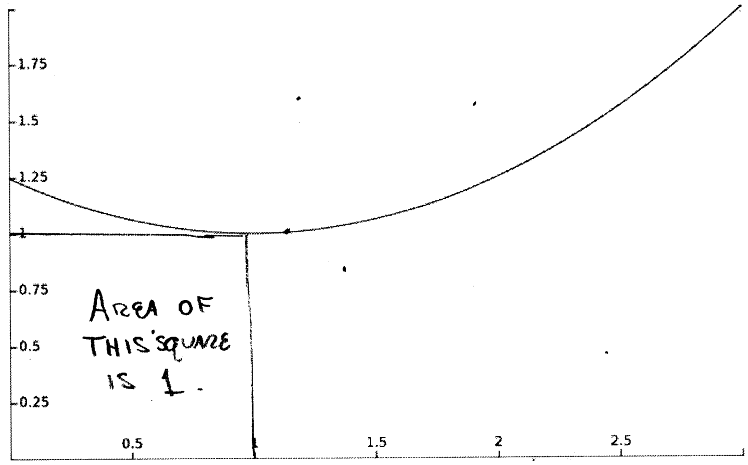
$\Rightarrow$ YES, 101 IS UNUSUAL.

(d) In January 2011, there were 67 traffic fatalities. Is this an unusually small number of fatalities?

$$\mu - 2\sigma = 79 - 2\sqrt{79} \approx 61.22$$

$\Rightarrow$ 67 IS NOT UNUSUAL.

7. (3 points) Explain why the graph shown below cannot be a probability density curve.



THE AREA UNDER THE CURVE IS GREATER THAN 1.

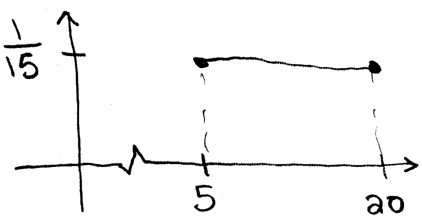
8. (8 points) A computer program generates random real numbers that are uniformly distributed between 5 and 20.

(a) If 1000 numbers were generated, would you expect more of them to be between 5 and 13 or between 13 and 20? Explain.

$13 - 5 = 8$ $\quad 20 - 13 = 7 \implies$ MORE WILL OCCUR IN THE LONGER INTERVAL ... BETWEEN 5 & 13.

(b) About what percent of the generated numbers will be between 11 and 17?

$20-5$
$=15$



$(17-11)\left(\frac{1}{15}\right) = \frac{6}{15} = 0.4$

$\boxed{40\%}$

9. (6 points) Suppose x is random variable in the normal distribution with mean 13.1 and standard deviation 2.7. Compute each of the following.

(a) $P(x = 15.0)$ $= 0$

(b) $P(x > 14.1)$ $=$ normalcdf(14.1, 999999, 13.1, 2.7)

$\approx 0.3556 = \boxed{35.56\%}$

(c) $P(x \geq 14.1)$

$=$ SAME AS (b)

$\approx \boxed{35.56\%}$

10. (9 points) The numbers of calories in 1.5-ounce chocolate bars are normally distributed with mean 225 and standard deviation 9.

(a) What is the probability that a randomly selected chocolate bar has between 220 and 230 calories?

$$P(220 \le x \le 230)$$
$$= \text{normalcdf}(220, 230, 225, 9)$$
$$\approx 0.4215 = \boxed{42.15\%}$$

(b) In a sample of 20 chocolate bars, about how many will contain fewer than 235 calories?

$$20 \cdot P(x < 235) = 20 \cdot \text{normalcdf}(-999999, 235, 225, 9)$$
$$\approx 17.33$$
$$\boxed{\text{ABOUT } 17}$$

(c) What is the probability that a randomly selected chocolate bar has fewer than 210 or more than 240 calories?

$$P(x < 210) + P(x > 240)$$
$$= 1 - \text{normalcdf}(210, 240, 225, 9)$$
$$\approx 0.0956 = \boxed{9.56\%}$$

11. (4 points) Patient recovery times after a certain medical procedure are normally distributed with mean 6.3 days and standard deviation 2.4 days. If a recovery time exceeds the 85th percentile, the patient is given an extensive follow-up examination. What recovery time is at the 85th percentile?

$$\text{invNorm}(0.85, 6.3, 2.4)$$
$$\approx \boxed{8.8 \text{ DAYS}}$$

12. (9 points) For each situation, decide whether the distribution of sample means is approximately normal. If so, find the mean and standard deviation of the sampling distribution.

(a) An automobile dealer finds that used car prices are normally distributed with mean \$15,700 and standard deviation \$2150. The dealer selects random samples of 15 cars.

YES, BECAUSE ORIGINAL POP. IS NORMAL.

$\mu_{\bar{x}} = 15700$

$\sigma_{\bar{x}} = \dfrac{2150}{\sqrt{15}} \approx 555.13$

(b) Tarsus lengths of adult male grackles have mean 33.80 mm and standard deviation 4.84 mm. A wildlife biologist collects random samples of size 32.

YES, BECAUSE $n > 30$.

$\mu_{\bar{x}} = 33.80$ mm

$\sigma_{\bar{x}} = \dfrac{4.84}{\sqrt{32}}$ mm ≈ 0.86 mm

(c) The finishing times in the New York City 10-km run had mean 61 minutes and standard deviation 9 minutes. Random samples of size 12 were obtained.

NO, ORIGINAL POP. NOT NORMAL & n IS TOO SMALL.

13. (9 points) On average, an American uses 123 gallons of water daily, with a standard deviation of 21 gallons. Thirty-five Americans are randomly selected.

(a) What is the probability that the mean water usage of the sample is between 120 gallons and 123 gallons?

normalcdf$(120, 123, 123, 21/\sqrt{35})$

$\approx 0.3010 = 30.10\%$

(b) What would be an unusually small sample mean?

$\mu_{\bar{x}} - 2\sigma_{\bar{x}} = 123 - 2\left(\dfrac{21}{\sqrt{35}}\right)$

≈ 115.9 GALLONS

(c) Are the sampling means normally distributed? Explain?

YES, BECAUSE $n = 35 > 30$.

14. (7 points) The sampling distribution for a sample statistic t is shown below.

t	1	3	5	5.5	7.5
$P(t)$	0.20	0.40	0.07	0.20	0.13

(a) Compute the mean of the sampling distribution.

$$\mu_t = 1(0.20) + 3(0.40) + 5(0.07) + 5.5(0.20) + 7.5(0.13) = \boxed{3.825}$$

(b) Suppose that the corresponding population parameter is 3.6. Do the sample statistics target the population parameter? Explain.

No, $3.825 \neq 3.6$

(c) Is this particular statistic a biased or an unbiased estimator? Explain.

BIASED, SAMPLE STATS DO NOT TARGET POP. PARAMETER.