Math 153 - Final Exam

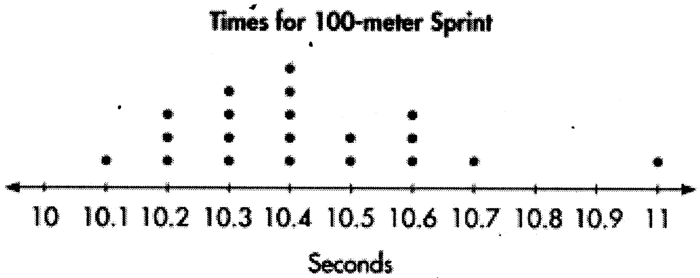
May 18, 2017

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) The graphic shown below gives the times, in seconds, for the 100-meter sprints of a number of runners.



(a) What is the name of this type of graphical display?

DOT PLOT

(b) How many data values are shown in the graph?

20

(c) What is the median time for running the 100-meter sprint? Give units with your answer.

$$\frac{10^{TH} + 11^{TH}}{2} = \boxed{10.4 \text{ SECONDS}}$$

(d) Based only on the distribution of data, which do you think is a better measure of center, the mean or the median? Briefly say why you think so.

MEDIAN, THE EXTREME VALUE AT THE RIGHT WILL PULL UP THE MEAN.

2. (10 points) Two different tests of battery life were performed on a certain brand of batteries. In one test, battery failure occurred after an average of 642.6 pulses with a standard deviation of 8.4 pulses. In the other test, battery failure occurred after an average of 8.76 hours with a standard deviation of 0.05 hours.

   (a) Compute the CV's. Which has greater spread, life in pulses or life in hours?

Pulses: ★

$$\frac{8.4}{642.6} \approx 1.31\%$$

Hours:

$$\frac{0.05}{8.76} \approx 0.57\%$$

Pulses have greater spread.

   (b) A particular battery had a lifetime of 8.85 hours. Compute the corresponding z-score.

$$z = \frac{8.85 - 8.76}{0.05} = \frac{0.09}{0.05} = \boxed{1.8}$$

   (c) Based on the z-score computed above, did that particular battery have an unusually long life? Briefly explain.

No, $1.8 < 2$.

3. (10 points) The Insurance Institute for Highway Safety conducted tests with crashes of new cars traveling at 6 mi/h. The costs of damage from a simple random sample are shown below. Compute the sample mean and standard deviation. Based on your result, is damage of $10,000 unusual? Briefly explain.

$7448 $4911 $9051 $6374 $4277

Calculator...

$\bar{X} = \$6412.2$

$S = \$1926.8$

$\bar{X} + 2s = \$10265.8$

$\Rightarrow$ $10000 is not unusual.

4. (10 points) The frequency distribution shown below gives the daily high temperatures (in °F) last year in Cleveland, OH.

High Temp (°F)	Frequency
20–30	19
31–41	43
42–52	68
53–63	69
64–74	74
75–85	68
86–96	24

(a) What is the class width?

$31 - 20 = 11$

(b) What are the class midpoints?

$\frac{20+30}{2} = 25, 36, 47, 58, 69, 80, 91$

(c) Use class midpoints to estimate the (weighted) mean temperature.

$$\frac{19(25) + 43(36) + \cdots + 68(80) + 24(91)}{19 + 43 + 68 + \cdots + 24}$$

$$= \frac{21951}{365} \approx 60.14°$$

5. (8 points) The following data the numbers of chocolate chips counted in each of 24 Hannaford Chocolate Chip cookies.

11 12 12 12 13 13 13 13 14 14 14 14
14 14 15 15 15 15 16 16 16 16 17 21

Determine the quartiles, the interquartile range, and the cutoff values for outliers.

$\text{Med} = \frac{14+14}{2} = 14$

$Q_1 = \frac{13+13}{2} = 13$

$Q_3 = \frac{15+16}{2} = 15.5$

$IQR = 15.5 - 13$

$= 2.5$

Cutoffs:

$13 - 1.5 \cdot 2.5 = 9.25$

$15.5 + 1.5 \cdot 2.5 = 19.25$

6. (12 points) The number of hits on a certain website follows a Poisson distribution with an average of 3 hits per minute.

   (a) What is the average number of hits per hour?

   $3 \times 60 = 180$ HITS/HOUR

   (b) In any given hour, what is the probability that the website is hit exactly 200 times?

   $P(x = 200) = \text{poissonpdf}(180, 200) \approx 0.0097$

   (c) In a given hour, what is the probability that the website is hit fewer than 170 times?

   $P(x < 170) = \text{poissoncdf}(180, 169)$

   ≈ 0.2183

7. (12 points) When women were allowed by the U.S. military to become fighter pilots, engineers needed to redesign the jets' ejection sets. Assume that weights of women are normally distributed with mean 165.0 lb and standard deviation 45.6 lb.

   (a) If a woman is randomly selected, find the probability that her weight is between 140 lb and 211 lb.

   $P(140 < x < 211) = \text{normalcdf}(140, 211, 165, 45.6)$

   ≈ 0.5517

   (b) If 36 different women are randomly selected, find the probability that their mean weight is between 140 lb and 211 lb.

   $P(140 < \bar{x} < 211) = \text{normalcdf}\left(140, 211, 165, \frac{45.6}{\sqrt{36}}\right)$

   ≈ 0.9995

   (c) For engineers redesigning ejection seats, which probability is more relevant: the result from part (a) or the result from part (b)? Briefly explain why.

   (a) --- EJECTION SEATS OPERATE FOR INDIVIDUALS, NOT GROUPS.

8. (16 points) A homeowner's association has a four-member board of directors. The ages of the directors are 36, 52, 57, and 62. Samples of two ages are randomly selected **with replacement**. Let x represent the range of a two-age sample.

(a) List all 16 samples as ordered pairs, e.g., $(57, 36)$.

(36,36) -- 0	(52,36) -- 16	(57,36) -- 21	(62,36) -- 26
(36,52) --- 16	(52,52) -- 0	(57,52) -- 5	(62,52) -- 10
(36,57) – 21	(52,57) -- 5	(57,57) -- 0	(62,57) -- 5
(36,62) --- 26	(52,62) -- 10	(57,62) -- 5	(62,62) - 0

(b) List all possible values of the random variable x? (Remember that x represents a sample range.)

0, 5, 10, 16, 21, 26

(c) Determine the probability distribution for the random variable x. Give your distribution in the form of a table.

X	0	5	10	16	21	26
P(x)	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$

(d) Find the mean value of x.

$$\mu = 0\left(\frac{4}{16}\right) + 5\left(\frac{4}{16}\right) + 10\left(\frac{2}{16}\right) + 16\left(\frac{2}{16}\right) + 21\left(\frac{2}{16}\right) + 26\left(\frac{2}{16}\right)$$

$$= \frac{20 + 20 + 32 + 42 + 52}{16} = 10.375$$

(e) Find the population range. Do the sample ranges target the population range? Explain.

$62 - 36 = 26$

No, $26 \neq 10.375$

9. (16 points) According to a recent Gallup poll, 4.1% of U.S. adults identify as LGBT. Forty-five U.S. adults are selected at random.

Binomial

$p = 0.041$

$n = 45$

$q = 0.959$

(a) What is the probability that exactly 3 people in the sample identify as LGBT?

$$P(x=3) = \text{binompdf}(45, 0.041, 3)$$

$$\approx 0.1685$$

(b) What is the probability that 4 or more identify as LGBT?

$$P(x \geq 4) = 1 - P(x \leq 3)$$

$$= 1 - \text{binomcdf}(45, 0.041, 3)$$

$$\approx \boxed{0.1120}$$

(c) In the sample of 45, what would be an unusually large number of people identifying as LGBT?

$$\mu + 2\sigma = np + 2\sqrt{npq}$$

$$\approx 4.5 \Rightarrow 5 \text{ people}$$

10. (12 points) A sociologist sampled 200 people who work in computer-related jobs and found that 11 of them have changed jobs in the past 6 months. Find a 95% confidence interval estimate for the proportion of those who work in computer-related jobs who have changed jobs in the past 6 months. Write a complete sentence that gives a valid interpretation of your interval.

1-PropZInt

$X = 11$

$n = 200$

C-Level $= 0.95$

$(0.0234, 0.0866)$

We are 95% that the true proportion of those who haved changed jobs is between 2.34% and 8.66%.

11. (16 points) The following data are the monthly dollar amounts for landline telephone service for a random sample of 8 small businesses in your community.

89, 47, 116, 58, 72, 101, 85, 105

You believe that, on average, local small businesses spend less than \$95 per month on landline phone service. (Assume that the service costs are normally distributed.)

(a) State the null and alternative hypotheses.

CLAIM: $\mu < 95$

COUNTER: $\mu \geq 95$

$\boxed{H_0 : \mu = 95 \quad H_1 : \mu < 95}$

(b) In order to test the claim, will you use a t-test or a z-test? Why?

BECAUSE σ IS UNKNOWN.

(c) Compute the test statistic.

CALCULATOR:
T-Test w/ Data
μ_0: 95.

$\boxed{t = -1.29178}$

(d) Find the P-value and draw a conclusion about your original claim at the level $\alpha = 0.10$. Write your conclusion in a complete sentence.

$\boxed{\text{P-VALUE} = 0.1187}$

SINCE P-VALUE $> \alpha = 0.10$,

WE DO NOT REJECT H_0.

AT THE LEVEL $\alpha = 0.10$,
THE EVIDENCE DOES NOT SUPPORT
THE ORIGINAL CLAIM.

12. (16 points) A production manager has compared employees' dexterity test scores to their hourly productivity.

Test score, x	Hourly units produced, y
12	55
14	63
17	67
16	70
11	51

(a) Compute the linear correlation coefficient, r, and use it to draw a conclusion about the strength of the linear relationship between x and y.

$r \approx 0.9546$

THERE IS A VERY STRONG LINEAR RELATIONSHIP BETWEEN TEST SCORE AND PRODUCTIVITY.

(b) Find the regression equation.

$\hat{y} = 3x + 19.2$

(c) Use your regression equation to predict the productivity of an employee whose test score is 19.

WHEN $x = 19$, $\hat{y} = 3(19) + 19.2$

$= 76.2$

(d) Predict the test score of an employee who produces 45 units per hour.

$45 = 3x + 19.2$

$\Rightarrow 3x = 25.8$

$x = 8.6$