Math 153 - Test 2
March 8, 2018

Name key

Score _____

Show all work to receive full credit. Supply explanations where necessary. You may use your calculator for all statistical computations.

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1. (6 points) When he was president, Barack Obama had a net worth of \$3,670,505. Including him, the 17 members of his executive branch had a mean net worth of \$4,939,455 with a standard deviation of \$7,775,948.

   (a) Compute the z-score for Obama's net worth.

   $$z = \frac{3670505 - 4939455}{7775948} \approx -0.16$$

   (b) Was Obama's net worth unusual among the executive branch? Explain.

   No, unusual values have z-scores greater than 2 or less than −2.

   (c) Give the net worth of a person whose worth is only a little bit unusual. Show work or explain.

   "Little bit" bigger than 2.

   $$X = 4939455 + 2.1(7775948)$$
   $$\approx \$21,268,946$$

2. (6 points) The following are the durations (in seconds) of 25 eruptions of Yellowstone's Old Faithful geyser. The data are in numerical order.

110	120	178	213	234	234	235	237	240	243
245	245	250	250	251	252	254	255	255	259
260	262	266	269	273					

   (a) What value is at the 30th percentile?

   $$\frac{L}{25} = 0.30 \Rightarrow L = 7.5 \Rightarrow$$ 8th value is 237 seconds

   (b) What value is at the 60th percentile?

   $$\frac{L}{25} = 0.60 \Rightarrow L = 15 \Rightarrow \frac{15^{TH} + 16^{TH}}{2} = \frac{251 + 252}{2}$$
   $$= 251.5 \text{ seconds}$$

3. (10 points) Survey participants were asked how much time they spend on morning hygiene and grooming. The following are their responses (in minutes) in numerical order.

LOWER HALF | UPPER HALF

4, 6, 7, 9, (14) 15, 15, 16, 18, | 18, 25, 26, 30, (32) 41, 45, 55, 63 *

Determine the quartiles, the interquartile range, and the cutoff values for outliers. Then sketch the modified boxplot. (Do all work by hand, but you may check your work on your calculator.)

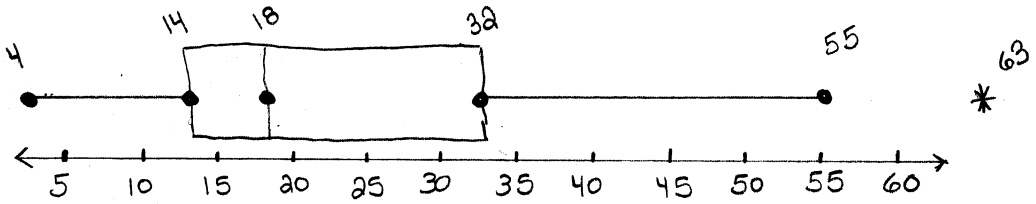
$\text{MED} = \frac{18+18}{2} = 18$

$Q_1 = 14$

$Q_3 = 32$

$\text{IQR} = 32 - 14 = 18$

CUTOFFS: $14 - 1.5(18) = -13$, $32 + 1.5(18) = 59$ $\Rightarrow$ 63 IS AN OUTLIER.



4. (3 points) Which of the following values are *NOT* probabilities? Circle all that apply.

(7 : 3) 3/7 (9/2) (−0.73) 0.123563 117/117 0 (56/24) 1

5. (3 points) In the 2014 Kentucky Derby, the odds against California Chrome, the winning horse, were 2 : 5. What was the probability of him winning the race?

ODDS IN FAVOR $= \frac{5}{2}$

$\text{PROB} = \frac{5}{2+5} = \boxed{\frac{5}{7}}$

6. (6 points) Four letters are selected from the word PROBABILITY. In each case, determine the probability of spelling BABY (in order).

(a) Letter selections are made without replacement.

$\frac{2}{11}$ B $\frac{1}{10}$ A $\frac{1}{9}$ B $\frac{1}{8}$ Y ----> $\boxed{\frac{2}{7920}}$

(b) Letter selections are made with replacement.

$\frac{2}{11}$ B $\frac{1}{11}$ A $\frac{2}{11}$ B $\frac{1}{11}$ Y ----> $\boxed{\frac{4}{14641}}$

7. (12 points) The table below shows the results from a company's pre-employment drug screening program.

	Positive drug test	Negative drug test	
Drug user	44	6	50
Not a drug user	90	860	950
	134	866	1000

One of the screening subjects is selected at random.

(a) What is the probability that the person tested positive?

$\frac{134}{1000} = 13.4\%$

(b) What is the probability that the person tested positive or is a drug user?

$\frac{90+44+6}{1000} = 14\%$

(c) What is the probability that the person tested positive and is a drug user?

$\frac{44}{1000} = 4.4\%$

(d) What is the probability that the person is a drug user given that he/she tested positive?

$\frac{44}{134} \approx 32.8\%$

(e) What is the probability that the person tested positive given that he/she is a drug user?

$\frac{44}{50} = 88\%$

(f) Are testing positive and being a drug user independent events? Use probabilities to support your answer.

No BECAUSE PROB OF POS TEST = 13.4% (From a)

IS NOT EQUAL TO

PROB OF POS TEST GIVEN USER = 88% (From e)

8. (4 points) American adults are selected for a Gallup poll. Let M be the event of selecting a male, and let R be the event of selecting a Republican. Are the events M and R disjoint (mutually exclusive)? Explain.

No, there are male Republicans.

9. (6 points) About 24% of Americans identify as Republicans. As above, let R be the event of randomly selecting a Republican from American adults.

   (a) What does $\overline{R}$ represent?

$\overline{R}$ = event of selecting an American adult who is not Republican.

   (b) What is a reasonable approximation for $P(\overline{R})$?

$P(\overline{R}) = 1 - 0.24 = 0.76 = 76\%$

   (c) About 10% of Americans are left-handed. Estimate the probability that a randomly selected American adult is a left-handed Republican.

Assuming Republican & left-handed are independent...

Prob is $0.10 \times 0.24 = 0.024 = 2.4\%$

10. (6 points) A month is randomly selected, ripped from a calendar, and discarded. Let J be the event that the selected month starts with the letter J. Let N be the event that the selected month is November. Are J and N independent? Show work to support your answer.

$P(N) = \frac{1}{12}$ $\quad P(N \mid J) = 0$

$\frac{1}{12} \neq \frac{0}{4}$ Not independent

11. (12 points) Suppose A and B are events such that $P(\overline{A}) = 0.62$, $P(B) = 0.67$, and $P(A \cap B) = 0.22$.

(a) Compute $P(A)$.

$1 - 0.62 = \boxed{0.38}$

(b) Compute $P(A \cap \overline{A})$.

$A \cap \overline{A}$ is impossible $\quad P(A \cap \overline{A}) = \boxed{0}$

(c) Compute $P(A \cup B)$.

$0.38 + 0.67 - 0.22 = \boxed{0.83}$

(d) Compute $P(B|A)$.

$$\frac{P(A \cap B)}{P(A)} = \frac{0.22}{0.38} \approx \boxed{0.579}$$

(e) Are A and B independent? Explain.

No, $P(B|A) \neq P(B)$

(d) (given)

(f) What are the odds in favor of B?

$P(B) = 0.67 \Rightarrow$ Odds $= \frac{67}{100-67} = \boxed{\frac{67}{33}}$

12. (12 points) The probability distribution for the random variable x is shown below.

x	1	2	3	4	5
$P(x)$	0.03	0.16	0.47	0.31	0.03

(a) What two things about the table above show that it is a probability distribution?

① EACH $P(x)$ IS A PROBABILITY. $0 \le P(x) \le 1$

② $\sum P(x) = 0.03 + 0.16 + 0.47 + 0.31 + 0.03 = 1$

(b) What is the mean value of x?

$\mu = 1(0.03) + 2(0.16) + 3(0.47) + 4(0.31) + 5(0.03)$

$= \boxed{3.15}$

(c) What is the standard deviation in x?

$\sigma^2 = 1(0.03) + 4(0.16) + 9(0.47) + 16(0.31) + 25(0.03) - (3.15)^2$

$= 0.6875$

$\sigma = \sqrt{0.6875} \approx \boxed{0.83}$

(d) Determine all unusual values of x. Show work or explain.

$\mu - 2\sigma \approx 1.49$

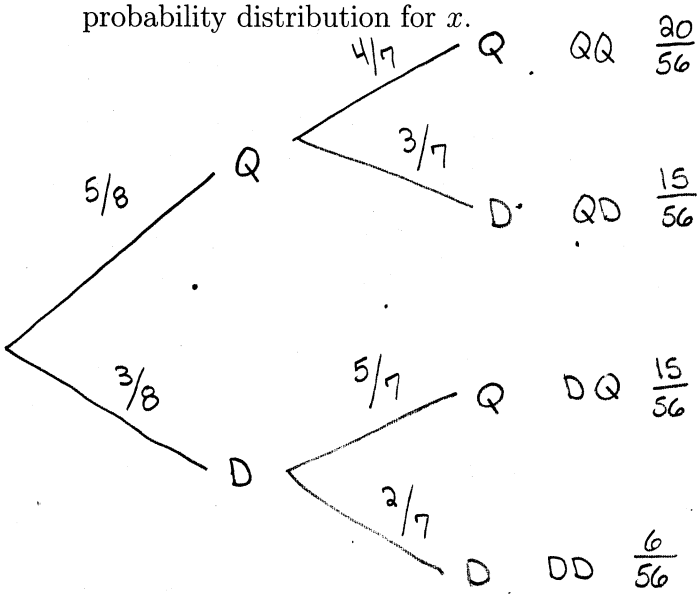
$\mu + 2\sigma \approx 4.81$

} 1 IS UNUSUALLY SMALL

5 IS UNUSUALLY LARGE

13. (8 points) A jar contains 5 quarters and 3 dimes. Two coins are selected without replacement. Let x represent the value (in cents) of the pair of coins. Determine the probability distribution for x.

POSSIBLE VALUES ARE $X = 20, 35, 50$



X	$P(x)$
20	$6/56$
35	$30/56$
50	$20/56$

14. (6 points) In a Prince Market Research survey of 2518 motorists, 252 said they made an obscene gesture in the previous month. $\Rightarrow$ 2266 DID NOT

(a) If two of the surveyed motorists are randomly selected without replacement, what is the probability that both did NOT make an obscene gesture?

$$\frac{2266}{2518} \times \frac{2265}{2517} = \frac{5132490}{6337806} \approx \boxed{80.98\%}$$

(b) If 20 of the surveyed motorists are randomly selected without replacement, what is the probability that none of them made an obscene gesture? (Hint: Estimate the probability using the fact that 20 is fewer than 5% of the population.)

WE CAN APPROXIMATE BY USING "WITH REPLACEMENT"

$$\left(\frac{2266}{2518}\right)^{20} \approx \boxed{12.14\%}$$