Math 153 - Test 3

April 19, 2018

Name key

Score ____________

Show all work to receive full credit. Supply explanations where necessary. You may use your calculator for all statistical computations, but show how you use it.

Normal
$\mu = 17.9$
$\sigma = 0.9$

1. (8 points) When in use, the batteries of a certain kind of electronic device have lives that are normally distributed with mean 17.9 hr and standard deviation 0.9 hr.

   (a) If a battery is selected at random, what is the probability that its life will exceed 20 hr?

   $$P(x > 20) = \text{normalcdf}(20, 999999, 17.9, 0.9)$$

   $$\approx \boxed{0.0098}$$

   (b) In a sample of 250 batteries, about how many will have lifetimes between 17 hr and 19 hr?

   $$250 \cdot P(17 \le x \le 19) = 250 \cdot \text{normalcdf}(17, 19, 17.9, 0.9)$$

   $$\approx 182.6 \qquad \boxed{\text{About } 183}$$

Binomial
$n = 50$
$p = 0.37$
$q = 0.63$

2. (12 points) According to a recent poll by the Pew Research Center, 37% of Americans have "some" trust in the news they obtain from social media. A random sample of 50 Americans is obtained.

   (a) What is the probability that exactly 20 have some trust in social media news?

   $$P(x = 20) = \text{binompdf}(50, 0.37, 20)$$

   $$\approx \boxed{0.1041}$$

   (b) What is the probability that fewer than 15 have some trust in social media news?

   $$P(x < 15) = P(x \le 14) = \text{binomcdf}(50, 0.37, 14)$$

   $$\approx \boxed{0.1195}$$

   (c) In the sample of 50, what would be an unusually large number of people who have some trust in social media news?

   $$\mu + 2\sigma = np + 2\sqrt{npq} \approx 25.33$$

   $$\boxed{26 \text{ or more}}$$

3. (9 points) The probability distribution for the random variable x is shown below.

x	1	2	3	4	5	6	7	8
$P(x)$	0.03	0.03	0.15	0.02	0.45	0.28	0.03	0.01

Using 5% rule

(a) Determine all unusually small values of x. Show work or explain.

$P(x \le 1) = 0.03$

$P(x \le 2) = 0.06$

$\Rightarrow$ 1 IS UNUSUALLY SMALL

(b) Determine all unusually large values of x. Show word of explain.

$P(x \ge 8) = 0.01$

$P(x \ge 7) = 0.04$

$P(x \ge 6) = 0.32$

$\Rightarrow$ 7 & 8 ARE UNUSUALLY LARGE

(c) Is $x = 4$ unusual? Explain.

No, $x = 4$ IS NOT EVEN IN A TAIL.

UNUSUAL VALUES ARE ONLY IN TAILS.

4. (12 points) A company is investigating the HVAC system in one of its out-buildings. On average, the building's furnace comes on 17 times per day.

Poisson $\mu = \lambda = 17$

(a) In a given day, what is the probability that the building's furnace comes on more than 20 times?

$P(x > 20) = 1 - P(x \le 20)$

$= 1 - \text{poissoncdf}(17, 20) \approx 0.1945$

(b) In a given day, what is the probability that the building's furnace comes on exactly 17 times?

$P(x = 17) = \text{poissonpdf}(17, 17) \approx 0.0963$

(c) What is the standard deviation in the numbers of times the furnace comes on per day?

$\sigma = \sqrt{\mu} = \sqrt{17} \approx 4.1$

5. (14 points) A three-person employee committee is exploring tech options for a small company. The ages of the committee members, in years, are 26, 32, and 62. In this problem, we analyze whether sampling is a good way to find a population maximum value. We take samples of size 2.

(a) List all possible samples of two ages, and state the maximum age in each sample.

(26, 26) --- 26   (32, 26) --- 32   (62, 26) --- 62
(26, 32) --- 32   (32, 32) --- 32   (62, 32) --- 62
(26, 62) --- 62   (32, 62) --- 62   (62, 62) --- 62

(b) Find the probability distribution for the sample maximums. Write your distribution in the form of a table.

X	P(x)
26	1/9
32	3/9
62	5/9

(c) Find the mean of the sampling distribution of the sample maximums.

$$\mu = 26\left(\frac{1}{9}\right) + 32\left(\frac{3}{9}\right) + 62\left(\frac{5}{9}\right) = \frac{432}{9} = \boxed{48}$$

(d) Look back at the original population of ages. Find the population maximum.

$$\max\{26, 32, 62\} = \boxed{62}$$

(e) Do sample maximums target the population maximum? Explain.

No, 48 = MEAN OF SAMPLING DISTRIBUTION
$\neq$ 62 = POP. MAX.

(f) Is sampling a good way to find a population maximum value? Explain.

No way. MAX VALUES ARE BIASED ESTIMATORS.
(SEE (e)).

6. (2 points) Suppose x is a random variable in a probability distribution. Which one of these must be true if $x = k$ is unusually small? (Circle one choice.)

   (a) $P(x = k) < 0.05$
   (b) $P(x \leq k) \geq 0.05$
   (c) $P(x \geq k) \leq 0.05$
   (d) $P(x = k) > 0.05$
   (e) $P(x \leq k) \leq 0.05$

7. (2 points) Suppose x is a random variable in a probability distribution. Which one of these must be true if $x = k$ is unusually large? (Circle one choice.)

   (a) $P(x = k) < 0.05$
   (b) $P(x \leq k) \geq 0.05$
   (c) $P(x \geq k) \leq 0.05$
   (d) $P(x = k) > 0.05$
   (e) $P(x \leq k) \leq 0.05$

8. (8 points) In a hospital emergency room, the processing times for arriving patients are normally distributed with mean 17 min and standard deviation 9 min. A random sample of 47 patients is obtained.

   (a) What is the probability that the mean processing time is less than 16 min?

   CLT 1 & 2 APPLY. $n = 47$

   $P(\bar{x} < 16) = \text{normalcdf}(-999999, 16, 17, 9/\sqrt{47})$
   $\approx \boxed{0.2231}$

   (b) Internal regulations require on-call employees to be called into work if the mean processing time reaches the 95th percentile. What mean processing time is at the 95th percentile?

   $\text{invNorm}(0.95, 17, 9/\sqrt{47}) \approx \boxed{19.16 \text{ MIN}}$

9. (9 points) For each situation, decide whether the distribution of sample means is approximately normal. If so, find the mean and standard deviation of the sampling distribution.

(a) Speeds on a highway are normally distributed with a mean of 71 mph and a standard deviation of 3 mph. 13 cars are picked at random and their speeds are measured.

Yes, BECAUSE THE ORIGINAL POP. IS NORMAL. CLT 2 APPLIES →

$\mu_{\bar{x}} = 71$ mph

$\sigma_{\bar{x}} = \frac{3}{\sqrt{13}}$ mph

(b) Adult male wombats in Narawntapu National Park have a mean weight of 35.6 kg with a standard deviation of 2.8 kg. Random samples of 20 male wombats are obtained.

No, NEITHER CLT 1 NOR CLT 2 APPLY.

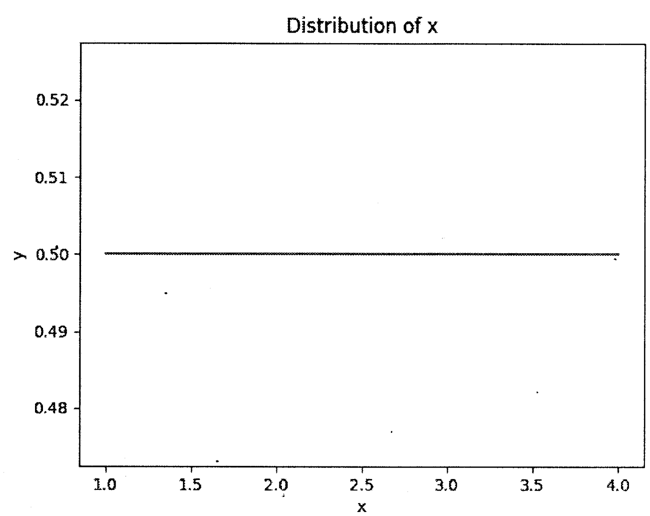
(c) Eggs have a mean cholesterol content of 215 mg with a standard deviation of 15 mg. Samples of 500 eggs are randomly selected.

Yes, CLT 1 APPLIES →

$\mu_{\bar{x}} = 215$ mg

$\sigma_{\bar{x}} = \frac{15}{\sqrt{500}}$ mg

10. (5 points) Is the graph shown below a probability density curve. Explain.

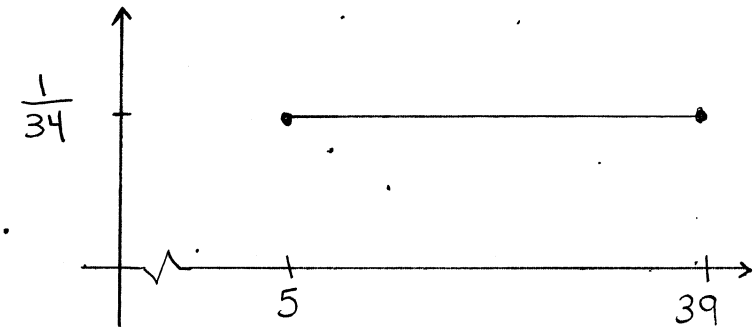


No, THE AREA UNDER THE CURVE $= \frac{1}{2}(4-1) = 1.5 \neq 1$

11. (9 points) In a certain large group of adults, the weekly amounts of time spent watching television are uniformly distributed between 5 hours and 39 hours.

(a) Sketch the density curve for the probability distribution.

$39 - 5 = 34$



(b) What is the probability that a random person from this group watches television more than 20 hours per week?

$$P(x > 20) = P(20 < x \le 39) = \frac{1}{34}(39 - 20)$$
$$= \boxed{\frac{19}{34} \approx 56\%}$$

(c) What is the probability that a random person from this group watches television exactly 20 hours per week?

THIS IS A CONTINUOUS DISTRIBUTION

$$P(x = 20) = 0$$

12. (5 points) On average, a certain school's cafeteria has 418 customers per day. It is probably NOT a good idea to think that people visiting a cafeteria is a Poisson process. Briefly explain why not.

CUSTOMERS ARE NOT UNIFORMLY DISTRIBUTED.

13. (5 points) According to data collected by the Pew Research Center, 19% of U.S. adults have absolutely no confidence that elected officials act in the best interests of the public. A random sample of 200 U.S. adults is obtained. Use guess and check to find the number of adults at 80th percentile. That is, find the number k so that $P(x \le k) = 0.80$.

binomcdf(200, 0.19, 42) $\approx$ 0.7932

binomcdf(200, 0.19, 43) $\approx$ 0.8394

LOOKS LIKE 42 IS THE 80TH PERCENTILE.