Math 153 - Final Exam

May 17, 2018

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (12 points) 125 Illinois hospitals were recently rated on a 1–10 scale according to the incidence of "hospital-acquired conditions," such as secondary infections. The higher the rating, the worse the score. The results are summarized below.

Rating	Frequency
1–1.9	3
2–2.9	12
3–3.9	16
4–4.9	23
5–5.9	23
6–6.9	21
7–7.9	17
8–8.9	5
9–9.9	5

125

(a) What are the class boundaries associated with the last class listed above?

8.95 & 9.95

(b) What is the class width?

$2-1 = \boxed{1}$

(c) What are the class midpoints?

$\frac{1+1.9}{2} = 1.45, 2.45, 3.45, \ldots, 9.45$

(d) Use class midpoints to estimate the (weighted) mean hospital rating.

$$\frac{1.45(3) + 2.45(12) + \cdots + 9.45(5)}{125} = \frac{668.25}{125} = 5.346$$

(e) Use class midpoints to estimate the (weighted) median.

63RD MIDPOINT = 5.45

2. (12 points) Survey participants were asked how much time they spend on morning hygiene and grooming. The numbers below are their responses, in minutes, listed in numerical order. Determine the quartiles, the interquartile range, and the boundary values for outliers. Then sketch the modified boxplot.

EXTREME   EXTREME

4, 6, 7, 9, 14, 15, 15, 16, 18, 18, 25, 26, 30, 32, 41, 45, 55, 63

Lower 1/2   Upper 1/2

$Q_1 = 14$

$\text{Med} = \frac{18+18}{2} = 18$

$Q_3 = 32$

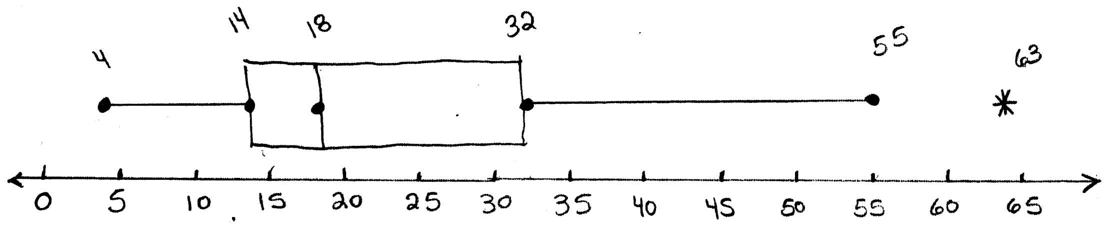
$IQR = 32 - 14 = 18$

BOUNDARY VALUES FOR OUTLIERS:

$14 - 1.5(18) = -13$

$32 + 1.5(18) = 59$

→ 63 IS THE ONLY OUTLIER



3. (8 points) Over the 25-year period from 1980 through 2004, the annual snowfall recorded at O'Hare Airport averaged 10.9 inches with a standard deviation of 7.5 inches. Over the same time period, Illinois averaged 41.0 tornadoes per year with a standard deviation of 28.3. Compute the CV's and determine whether inches of snowfall or numbers of tornadoes have greater relative spread.

SNOWFALL:

$CV = \frac{7.5}{10.9} \approx 68.8\%$

TORNADOES:

$CV = \frac{28.3}{41.0} \approx 69.0\%$

Very close! BUT TORNADOES HAVE A BIT MORE SPREAD.

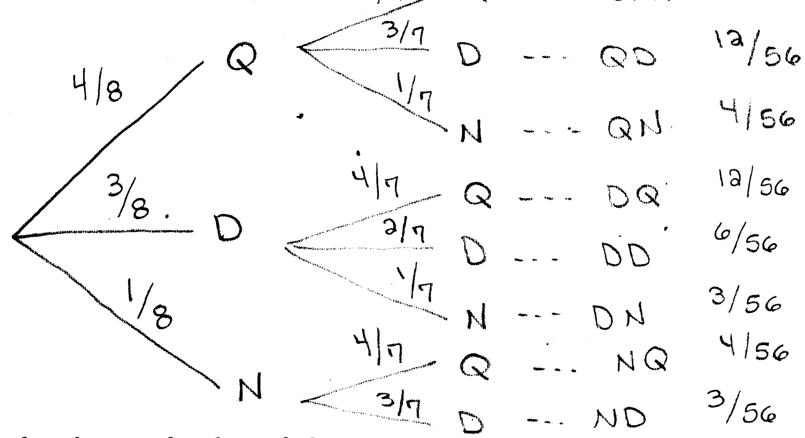
4. (6 points) Heights of men are normally distributed with mean 69 in and standard deviation 2.8 in. A contractor wishes to determine the height of a basement ceiling that allows 99.5% of all men to stand without crouching. What would that height be?

invNorm (0.995, 69, 2.8)

≈ 76.2 in

5. (16 points) A jar contains 4 quarters, 3 dimes, and 1 nickel. Two coins are selected at random (without replacement).

   (a) Sketch the complete tree diagram for this experiment. **Include the probabilities of each path.**



   (b) Let x be the total value of the two-coin sample in cents. List all possible values of x.

50¢, 35¢, 30¢, 20¢, 15¢

   (c) Use your tree diagram to determine the probability distribution for x. Give your answer in the form of a table.

X	15	20	30	35	50
P(x)	$\frac{6}{56}$	$\frac{6}{56}$	$\frac{8}{56}$	$\frac{24}{56}$	$\frac{12}{56}$

   (d) Find the mean value of x.

$$15\left(\frac{6}{56}\right) + 20\left(\frac{6}{56}\right) + 30\left(\frac{8}{56}\right) + 35\left(\frac{24}{56}\right) + 50\left(\frac{12}{56}\right)$$

$$= \frac{1890}{56} = 33.75¢$$

   (e) Based on your probability distribution, are there any unusual values of x. Explain how you know.

NO, THE LEAST INDIVIDUAL PROB IS $\frac{6}{56} \approx 10.7\%$ WHICH EXCEEDS 5%.

6. (12 points) Suppose A and B are events such that $P(\overline{A}) = 0.62$, $P(B) = 0.67$, and $P(A \cap B) = 0.22$.

(a) Compute $P(A)$. $1 - 0.62 = 0.38$

(b) Compute $P(A \cap \overline{A})$. $= P(\phi) = 0$

(c) Compute $P(A \cup B)$. $0.38 + 0.67 - 0.22 = 0.83$

(d) Compute $P(B|A)$. $\frac{P(A \cap B)}{P(A)} = \frac{0.22}{0.38} \approx 0.579$

(e) Are A and B independent? Explain.

No, $P(B|A) = 0.579 \neq P(B) = 0.67$

(f) What are the odds in favor of B?

$\frac{0.67}{0.33} = \frac{67}{33} \approx 2$ TO 1

7. (16 points) In a certain area, weekly SNAP benefits (per recipient) are normally distributed with mean \$115 and standard deviation \$22.

(a) A SNAP recipient is randomly selected. What is the probability that his weekly benefit is exceeds \$120?

$P(x > 120) = \text{normalcdf}(120, 999999, 115, 22) \approx 0.4101$

(b) If 26 SNAP recipients are are randomly selected, find the probability that the mean benefit exceeds \$118.

$P(\overline{x} > 118) = \text{normalcdf}(118, 999999, 115, 22/\sqrt{26}) \approx 0.2434$

(c) In samples of size 26, what would be an unusually large mean benefit?

$\mu_{\overline{x}} + 2\sigma_{\overline{x}} = 115 + 2(22/\sqrt{26}) \approx 123.63$

Bigger means would be unusual.

(d) Suppose SNAP benefits were not normally distributed. Would have this changed your approach to solving (b)? Briefly explain.

Yes, because the CLT would not have applied.

Sample is too small!

8. (12 points) 89% of U.S. adults use the internet. A random sample of 575 U.S. adults is obtained.

BINOMIAL

$p = 0.89$

$q = 0.11$

$n = 575$

(a) In samples of size 575, what is an unusually large number of adults who use the internet?

$$\mu + 2\sigma = np + 2\sqrt{npq} \approx 526.76$$

527 OR MORE

(b) What is the probability that at most 540 people in the sample use the internet?

$$P(X \leq 540) = \text{binomcdf}(575, 0.89, 540)$$

$$\approx 0.99998 \approx 1$$

(c) In the sample of 575 adults, what is the probability that exactly 511 or 512 adults use the internet?

$$P(X=511) + P(X=512) = \text{bimonpdf}(575, 0.89, 511) + \text{bimonpdf}(575, 0.89, 512)$$

$$\approx 0.1057$$

9. (12 points) Vehicles pass through a junction on a busy road at the rate of 300 per hour.

POISSON

$\lambda = \mu = 300$

(a) In any given hour, what is the probability that exactly 320 vehicles pass through the junction?

$$P(X=320) = \text{poissonpdf}(300, 320)$$

$$\approx 0.01161$$

(b) Terrible traffic back-ups occur if the rate exceeds 340 vehicles per hour. In any given hour, what is the probability that more than 340 vehicles pass through the junction?

$$P(X > 340) = 1 - P(X \leq 340)$$

$$= 1 - \text{poissoncdf}(300, 340)$$

$$\approx 0.01079$$

(c) In any given hour, what would be an unusually small number of vehicles passing through the junction?

$$\mu - 2\sigma = 300 - 2\sqrt{300} \approx 265.36$$

265 OR FEWER

0.67
× 2541
≈ 1702

10. (12 points) The Pew Research Center recently conducted a survey of 2541 randomly selected American adults. They found that 67% of survey respondents believe that the government is doing too little to reduce the effects of climate change.

(a) Compute a 95% confidence interval estimate for the true proportion of American adults who believe the government is doing too little to reduce climate change. State your conclusion in a complete sentence.

1-PropZInt

X = 1702

n = 2541

C-Level = 0.95

(0.65153, 0.6881)

WE ARE 95% CONFIDENT THAT THE TRUE MEAN IS BETWEEN 65.2% AND 68.8%

(b) What is the margin of error in your confidence interval estimate?

$$\frac{0.6881-0.65153}{2} \approx 0.018 = \boxed{1.8\%}$$

(c) Suppose the Pew Research Center is going to conduct the survey again, and they would like to predict the proportion with a margin of error no greater than 1%. What sample size should they use?

$$n = \frac{z_{\alpha/2}^2\, \hat{p}\hat{q}}{E^2} = \frac{(1.96)^2(0.67)(0.33)}{(0.01)^2} = 8493.78$$

USE n = 8494

11. (16 points) In a certain occupation, men earn a mean salary of \$40,668. The following data are the salaries (in \$1000s) of a random sample of women in the same occupation.

28.3, 30.5, 35.0, 37.2, 39.0, 30.4, 31.0, 29.8, 41.2, 28.9

You believe that, on average, women's salaries are less than men's, and you intend to test your hypothesis.

(a) State the null and alternative hypotheses.

$$\boxed{\begin{aligned} H_0 &: \mu = 40.668 \\ H_1 &: \mu < 40.668 \end{aligned}}$$

(b) In order to test the claim, will you use a *t*-test or a *z*-test? Why?

POP STD. DEV (σ) IS NOT KNOWN.

(c) In order to trust the conclusion of your hypothesis test, what assumption must be made?

THE POP. OF WOMEN'S SALARIES IS NORMALLY DISTRIBUTED.

(d) Compute the test statistic.

T-Test w/ Data μ_0: 40.668

$$\boxed{t = -5.176}$$

$\bar{X} = 33.13$

$S \approx 4.6$

(e) Find the *P*-value and draw a conclusion about your original claim at the level $\alpha = 0.05$. Write your conclusion in a complete sentence.

THE P-VALUE $\approx 2.9 \times 10^{-4} = 0.00029 < \alpha$

$\Rightarrow$ WE REJECT H_0.

THERE IS STRONG EVIDENCE IN SUPPORT OF THE CLAIM THAT $\mu < 40.668$.

12. (16 points) Data from a random sample of used Porsche sports cars were obtained from `Autotrader.com`.

Mileage (in thousands), x	Price (in \$1000s), y
16.7	95.8
18.2	91.7
32.7	45.7
26.2	72.0
23.4	79.5
41.9	65.0
54.1	41.0
73.2	30.1

(a) Compute the linear correlation coefficient, r, and use it to draw a conclusion about the strength (and direction) of the linear relationship between x and y.

$r \approx -0.89986$

≈ -0.9

THERE IS A STRONG NEGATIVE LINEAR CORRELATION.

(b) Find the regression equation.

$y \approx -1.107x + 104.729$

(c) Use your regression equation to predict the cost of a used Porsche with 60,000 miles.

$X = 60$

$\Rightarrow y = 38.3 \Rightarrow$ \$38,300

(d) If a used Porsche was selling for \$55,000, about how many miles would you predict are on it?

$y = 55$

$\Rightarrow 55 = -1.107x + 104.729$

$\Rightarrow x \approx 44.9 \Rightarrow$ 44,900 MILES