

Some population parameters are approximated well by sampling. Some are not. When a population parameter is equal to the mean of the distribution of its sample statistic, we say that the sample statistic targets the population parameter.

1. The ages, in years, of the four U.S. presidents when they were assassinated in office are 56 (Lincoln), 49 (Garfield), 58 (McKinley), and 46 (Kennedy). Samples of two ages are randomly selected **with replacement**. Let x represent the mean of a two-age sample.

(a) List all 16 samples as ordered pairs, e.g., (46, 56).

(56, 56) -- 56	(49, 56) -- 52.5	(58, 56) -- 57	(46, 56) -- 51
(56, 49) -- 52.5	(49, 49) -- 49	(58, 49) -- 53.5	(46, 49) -- 47.5
(56, 58) -- 57	(49, 58) -- 53.5	(58, 58) -- 58	(46, 58) -- 52
(56, 46) -- 51	(49, 46) -- 47.5	(58, 46) -- 52	(46, 46) -- 46

(b) List all possible values of the random variable x ? (Remember that x represents a sample mean.)

46, 47.5, 49, 51, 52, 52.5, 53.5, 56, 57, 58

(c) Determine the probability distribution for the random variable x . Give your distribution in the form of a table.

x	46	47.5	49	51	52	52.5	53.5	56	57	58
$P(x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

(d) Find the mean value of x .

$$\mu = 46\left(\frac{1}{16}\right) + 47.5\left(\frac{2}{16}\right) + \dots + 58\left(\frac{1}{16}\right) = \frac{836}{16} = 52.25$$

(e) Find the population mean. Do the sample means target the population mean?

$$\frac{56 + 49 + 58 + 46}{4} = \frac{209}{4} = 52.25$$

Yes!
Equal.

2. A homeowner's association has a four-member board of directors. The ages of the directors are 36, 52, 57, and 62. Samples of two ages are randomly selected **with replacement**. Let x represent the range of a two-age sample.

(a) List all 16 samples as ordered pairs, e.g., (57, 36).

$(36, 36) \rightarrow 0$	$(52, 36) \rightarrow 16$	$(57, 36) \rightarrow 21$	$(62, 36) \rightarrow 26$
$(36, 52) \rightarrow 16$	$(52, 52) \rightarrow 0$	$(57, 52) \rightarrow 5$	$(62, 52) \rightarrow 10$
$(36, 57) \rightarrow 21$	$(52, 57) \rightarrow 5$	$(57, 57) \rightarrow 0$	$(62, 57) \rightarrow 5$
$(36, 62) \rightarrow 26$	$(52, 62) \rightarrow 10$	$(57, 62) \rightarrow 5$	$(62, 62) \rightarrow 0$

(b) List all possible values of the random variable x ? (Remember that x represents a sample range.)

0, 5, 10, 16, 21, 26

(c) Determine the probability distribution for the random variable x . Give your distribution in the form of a table.

x	0	5	10	16	21	26
$P(x)$	$\frac{4}{16}$	$\frac{4}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$	$\frac{2}{16}$

(d) Find the mean value of x .

$$\mu = 0\left(\frac{4}{16}\right) + 5\left(\frac{4}{16}\right) + \dots + 26\left(\frac{2}{16}\right) = \frac{166}{16} = 10.375$$

(e) Find the population range. Do the sample ranges target the population range?

$$\text{Range} = 62 - 36 = 26$$

No, $10.375 \neq 26$.