Math 153 - Test 3
April 18, 2019

Name key

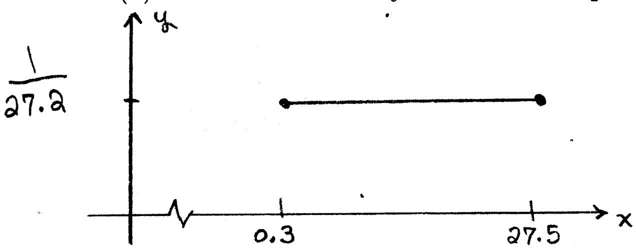
Score ________

Show all work to receive full credit. Supply explanations where necessary. You may use your calculator for all statistical computations, but show how you use it.

1. (9 points) A file server holds a large number of computer files. The files on the server are uniformly distributed in size from 0.3 MB to 27.5 MB.

   (a) Sketch the density curve for the probability distribution.

   $27.5 - 0.3 = 27.2$



   (b) What is the probability that a file selected at random will be smaller than 13.0 MB?

   $$P(x < 13) = (13-0.3)\left(\frac{1}{27.2}\right) = \frac{12.7}{27.2} \approx 0.4669$$

   (c) On the server, what file size is at the 90th percentile?

   $$(x-0.3)\left(\frac{1}{27.2}\right) = 0.90 \implies x - 0.3 = 24.48$$
   $$x = 24.78$$

2. (12 points) According to the Census Bureau, more than 30% of American children live with a single parent. Let's assume that the probability that a randomly selected child lives with a single parent is exactly 30%. Suppose that a simple random sample of 45 children is obtained.

   BINOMIAL
   $n = 45$
   $p = 0.30$
   $q = 0.70$

   (a) What is the probability that exactly 15 children live with a single parent?

   $$P(x=15) = \text{binompdf}(45, 0.30, 15) \approx 0.1115$$

   (b) What is the probability that more than 25 live with a single parent?

   $$P(x > 25) = 1 - P(x \le 25) = 1 - \text{binomcdf}(45, 0.30, 25) \approx 9.973 \times 10^{-5}$$

   (c) In the sample of 45 children, what would be an unusually small number of children who live with a single parent. Show work to justify your answer.

   $$\mu - 2\sigma = np - 2\sqrt{npq}$$
   $$= (45)(0.3) - 2\sqrt{(45)(0.3)(0.7)} \approx 7.35$$

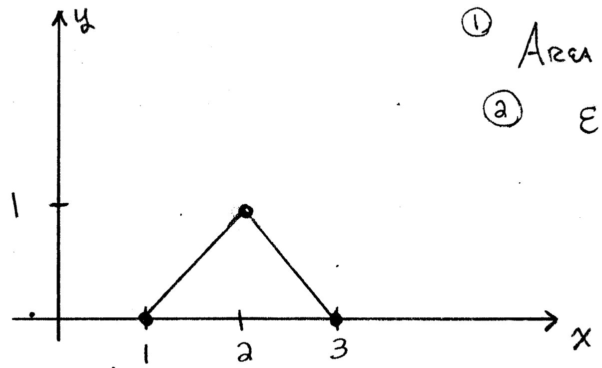
   7 OR FEWER IS UNUSUAL.

3. (2 points) Suppose x is a random variable in a binomial distribution. Also suppose that $P(x = 7) = 0.013$. Does this mean that 7 is an unusual x-value? Explain.

No, you must use a CDF to check for unusual values.

Look at $P(x \le 7)$ or $P(x \ge 7)$.

4. (5 points) A graph consists of two straight line segments connecting the points $(1, 0)$, $(2, 1)$ and $(3, 0)$, in that order. Is the graph a probability density curve. Explain.



① Area under curve $= \frac{1}{2}(3-1)(1) = 1$.

② Each y-coord ≥ 0

Yes, it is a density curve.

5. (5 points) For each situation, determine whether the random variable x is in a binomial distribution. If not, explain why.

(a) Let x be the number of calls received by the information center on a given day.

No. If the trials are calls received, then there is not a fixed number of trials. There is not really a success or failure either.

(b) A jar contains 50 Skittles candies. Ten candies are selected and eaten, one at a time. Let x be the number of red candies selected.

No. 10 trials are performed without replacement and population is small (50).

(c) A die is rolled nine times. Let x be the number of 6's that are rolled.

Yes! Nine trials are independent. Success = Roll 6.

Prob of success $= \frac{1}{6}$ = constant.

6. (8 points) The femur is the longest and largest bone in the human body. In adult males, lengths of femurs are approximately normally distributed with mean 48.0 cm and standard deviation 3.4 cm.

   (a) An adult male is selected at random. What is the probability that his femur is longer than 54 cm or shorter than 45 cm?

$$P(x>54) + P(x<45) = 1 - P(45<x<54)$$
$$= 1 - \text{normalcdf}(45, 54, 48, 3.4) \approx \boxed{0.2276}$$

   (b) We obtain a random sample of 75 adult men. About how many in the sample have femurs longer than 51.5 cm?

$$75 * P(x>51.5) = 75 * \text{normalcdf}(51.5, 999999, 48, 3.4)$$
$$\approx 11.37 \quad \boxed{\text{About } 11}$$

7. (10 points) The probability distribution for the random variable x is shown below.

x	2	3	4	5	6	7	8	9
$P(x)$	0.01	0.02	0.01	0.35	0.45	0.01	0.14	0.01

   (a) What two things about the table above show that it describes probability distribution?

   1. For each x, $0 \le P(x) \le 1$

   2. $\sum P(x) = 1$

   (b) Is 7 an unusual x-value?

No, $P(x \ge 7) = 0.16 > 5\%$

$P(x \le 7) = 0.85 > 5\%$

   (c) Determine all unusually small values of x. Show work or explain.

$\boxed{2, 3, 4}$ are unusually small because

$P(x \le 4) = 0.04 < 5\%$ (But 5 is not unusual. $P(x \le 5) = 0.39$)

   (d) Determine all unusually large values of x. Show word of explain.

9 is unusually large because

$P(x \ge 9) = 0.01 < 5\%$ (But 8 is not unusual. $P(x \ge 8) = 0.15$)

8. (9 points) For each situation, decide whether the distribution of sample means is approximately normal. If so, find the mean and standard deviation of the sampling distribution.

   (a) In studying adult male Brazilian tawny red tarantulas, Professor E.O. Wilson found that carapaces have a mean length of 18.14 mm with a standard deviation of 1.76 mm. Forty tarantulas are selected at random.

   Yes, CLT #1

   $$\mu_{\bar{x}} = 18.14$$

   $$\sigma_{\bar{x}} = \frac{1.76}{\sqrt{40}} \approx 0.28$$

   (b) The mean price of a TI-84 Plus graphing calculator is \$104.72, while the standard deviation in price is \$7.31. In studying office supply retailers across the country, 15 TI-84 Plus calculators are selected at random.

   SAMPLE SIZE TOO SMALL.

   POP. DISTRIBUTION UNKNOWN.

   CLT DOES NOT APPLY.

   (c) Heights of American men are normally distributed with mean 69.0 in and standard deviation 2.8 in. Random samples of 7 men are obtained.

   Yes, CLT #2

   $$\mu_{\bar{x}} = 69.0$$

   $$\sigma_{\bar{x}} = \frac{2.8}{\sqrt{7}} \approx 1.1$$

9. (12 points) Mode 1 Real Estate sells an average of 13 homes per month. Assume that monthly sales numbers are in a Poisson distribution.

   $\mu = 13$

   (a) In any given month, what is the probability that Mode 1 sells fewer than 10 homes?

   $$P(x < 10) = P(x \le 9) = \text{poissoncdf}(13, 9)$$

   $$= 0.1658$$

   (b) What is the probability that Mode 1 sells exactly 12 homes?

   $$P(x = 10) = \text{poissonpdf}(13, 12)$$

   $$= 0.1099$$

   (c) What is an unusually large number of sales in a month?

   $$\mu + 2\sigma = \mu + 2\sqrt{\mu} = 13 + 2\sqrt{13}$$

   $$\approx 20.2$$

   21 OR MORE

10. (14 points) A three-person committee has members with ages 29, 35, and 45. In this problem, we analyze whether sampling is a good way to find a population median. We take samples of size 2 **with replacement**.

(a) List all 9 samples of two ages, and state the median age of each sample.

(29, 29) -- 29　　(35, 29) -- 32　　(45, 29) -- 37

(29, 35) -- 32　　(35, 35) -- 35　　(45, 35) -- 40

(29, 45) -- 37　　(35, 45) -- 40　　(45, 45) -- 45

(b) Determine the probability distribution for the sample medians. Write your distribution in the form of a table.

X	29	32	35	37	40	45
P(x)	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(c) Find the mean of the sampling distribution of the sample medians.

$$\mu_{MED} = 29\left(\tfrac{1}{9}\right) + 32\left(\tfrac{2}{9}\right) + \dots + 45\left(\tfrac{1}{9}\right) = \frac{327}{9} = \boxed{36.\overline{3}}$$

(d) Look back at the original population of ages. Find the population median.

29, 35, 45

↓

$\boxed{\text{MED} = 35}$

(e) Do sample medians target the population median? Explain.

No, MED $= 35 \neq 36.\overline{3} = \mu_{MED}$

(f) Is the median a biased or unbiased estimator? Explain.

BIASED. SEE (e). SAMPLE MEDIANS DO NOT TARGET POP. MEDIAN.

11. (9 points) From 1962 to 1981, weights of U.S. pennies were approximately normally distributed with mean 3.11 g and standard deviation 0.029 g. A random sample of 16 pennies is obtained.

$\mu_{\bar{x}} = 3.11$

$\sigma_{\bar{x}} = \frac{0.029}{\sqrt{16}}$

(a) What is the probability that the sample mean is less than 3.07 g?

$$P(\bar{x} < 3.07) = \text{normalcdf}\left(-999999, 3.07, 3.11, \frac{0.029}{4}\right)$$

$$= \boxed{1.726 \times 10^{-8}} \approx 0$$

(b) What would be an usually small sample mean?

$$\mu_{\bar{x}} - 2\sigma_{\bar{x}} = 3.11 - 2\left(\frac{0.029}{\sqrt{16}}\right) = 3.0955$$

ANYTHING LESS THAN 3.0955 g

(c) What sample mean is at the 90th percentile?

$$\text{invNorm}\left(0.90, 3.11, 0.029/4\right)$$

$$= \boxed{3.1193 \text{ g}}$$

12. (5 points) Suppose x is a random variable in a probability distribution. Which of these would show that $x = 9$ is an unusual value? (Circle all that apply.)

(a) $P(x = 9) = 0.01$

(b) $P(x \leq 9) = 0.045$ By 5% RULE

(c) $P(8 \leq x \leq 11) = 0$

(d) $P(x \geq 9) = 0.03$ By 5% RULE

(e) $P(x \leq 9) = 0.23$