Math 153 - Final Exam

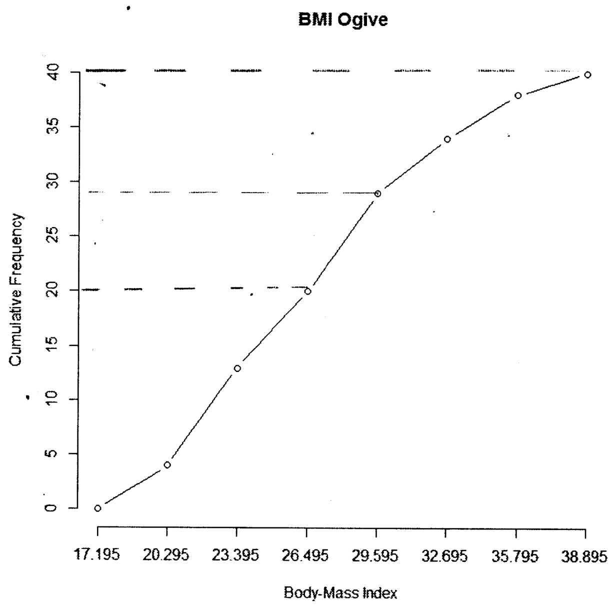
May 16, 2019

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (10 points) The following ogive summarizes the body mass indices (BMI's) of adult men from a random sample. (A larger copy of the ogive is included on the last page of the test.)



(a) How many men are in the sample?

$\boxed{40}$

(b) About how many men in the sample have BMI's between 26.495 and 29.595?

About $29 - 20 = \boxed{9}$

(c) In which interval of BMI's are there the fewest men?

Looks like least steep on $\boxed{35.795 - 38.895}$

(d) In which interval of BMI's are there the most men?

Steepest on $\boxed{20.295 - 23.395 \text{ or } 26.495 - 29.595}$

(e) Assuming the numbers along the horizontal axis are class boundaries, what are the class midpoints? (Just compute the first one.)

$$\frac{17.195 + 20.295}{2} = \boxed{18.745}$$

2. (5 points) Data from a sample have a CV (coefficient of variation) of 9.31% and a standard deviation of 48.28.

(a) Determine the sample mean.

$$CV = \frac{s}{\bar{x}} \Rightarrow \bar{x} = \frac{s}{CV} \qquad \bar{x} = \frac{48.28}{0.0931} \approx \boxed{518.6}$$

(b) Is the CV a measure of center, spread, position, or skewness?

SPREAD

3. (8 points) Here is a random sample of normal adult body temperatures (in °F):

99.1, 98.8, 98.4, 98.6, 98.2, 97.6, 98.0, 98.2, 97.5, 98.8, 97.8, 97.1.

Compute the mean and standard deviation. Based on the data, what are unusually low and high normal temperatures?

CALCULATOR...

$\boxed{\bar{x} = 98.175}$

$\boxed{s \approx 0.60}$

UNUSUAL:

HIGH: $\bar{x} + 2s \approx$ 99.4 AND ABOVE

LOW: $\bar{x} - 2s \approx$ 97.0 AND BELOW

4. (5 points) The salaries of the six employees at a local small business are shown below.

$42,350, $45,100, $43,000, $44,700 $225,125, $42,350

(a) Which is a more appropriate measure of center, the mean or the median? Why do you think so?

MEDIAN, BECAUSE THE EXTREME VALUE ($225,125) WILL PULL UP THE MEAN.

(b) Compute the measure of center that you chose in part (a).

42350, 42350, $\underbrace{43000, 44700}$, 45100, 225125

$$\text{MEDIAN} = \frac{43000 + 44700}{2} = \boxed{43,850}$$

5. (4 points) In the following display, 4 | 5 means 4.5.

3	1 6
4	1 2 5
5	0 0 2 6 7 8
6	3 8 8
7	0 4

(a) What is the name of this type of graphical display?

STEM-AND-LEAF PLOT

(b) **Without computing the mean or the median**, which one of these would you expect? Circle your choice and briefly explain.

- mean > median
- mean < median
- mean ≈ median

THE DISTRIBUTION IS SYMMETRIC, NOT SKEWED.

6. (6 points) The numbers shown below are the amounts of yearly snowfall, in inches, measured at O'Hare Airport in the years from 1968 to 1979.

10.4, 3.7, 9.5, 10.0, 7.6, 0.5, 7.4, 3.5, 10.0, 7.2, 21.9, 34.3

Compute the five-number summary, the IQR, and the outlier cutoff values.

CALCULATOR ...

$Q_1 = 5.45$

MED = 8.55

$Q_3 = 10.2$

MIN = 0.5

MAX = 34.3

$IQR = 10.2 - 5.45 = \boxed{4.75}$

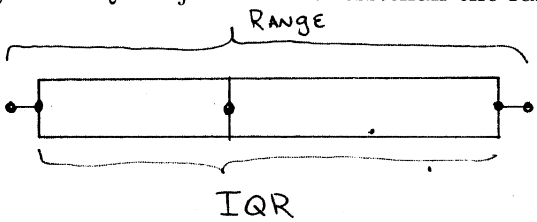
CUTOFFS:

$Q_1 - 1.5 \times IQR = \boxed{-1.675}$

$Q_3 + 1.5 \times IQR = \boxed{17.325}$

7. (4 points) Think carefully about the characteristics of a modified boxplot. For each part of this problem, sketch a boxplot that would correspond to a data set with the given properties.

   (a) The IQR is just a little less than the range.



   (b) There is one outlier in the lower extreme.

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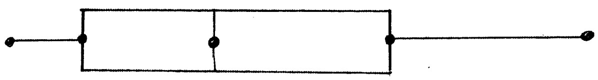
More than 1.5 IQR's below Q_1



8. (6 points) The values of the random variable x and its probabilities are shown below.

x	0	1	2	3	4	5	6
$P(x)$	0.03	?	0.41	0.21	0.29	?	0.01

Find possible values for $P(x = 1)$ and $P(x = 5)$ so that the table describes a probability distribution **and** both $x = 1$ and $x = 5$ are unusual.

Must have $P(x=1) + P(x=5) + 0.95 = 1 \Rightarrow P(x=1) + P(x=5) = 0.5$

$P(x=1) < 0.02$

$P(x=5) < 0.04$

Let $P(x=1) = 0.015$

$P(x=5) = 0.035$

9. (2 points) Prior to the running of the Kentucky Derby, the odds in favor of Maximum Security were 9:2. What was his probability of winning?

$$\text{Odds} = \frac{9}{2} \Rightarrow \text{Prob} = \frac{9}{9+2} = \boxed{\frac{9}{11}}$$

10. (4 points) The mean of 27 scores is 69.35. If two new scores, 92.5 and 87, are added to the collection, what is the new mean?

$$\frac{27(69.35) + 92.5 + 87}{29} = \frac{2051.95}{29} \approx \boxed{70.76}$$

11. (12 points) Participants in a clinical trial were separated according to biological sex and smoking status. The data are described in the table below.

	Female	Male	
Smoker	7	10	17
Nonsmoker	120	89	209
	127	99	226

A participant is selected at random.

(a) What is the probability that the person is a male?

$$\frac{99}{226} \approx 43.8\%$$

(b) What is the probability that the person is a smoker?

$$\frac{17}{226} \approx 7.5\%$$

(c) What is the probability that the person is a male or a smoker?

$$\frac{7+10+89}{226} = \frac{106}{226} \approx 46.9\%$$

(d) What is the probability that the person is a male, smoker?

$$\frac{10}{226} \approx 4.4\%$$

(e) What is the probability that the person is a smoker given that he is a male?

$$\frac{10}{99} \approx 10.1\%$$

(f) Are being a male and being a smoker independent events? Use probabilities to support your answer.

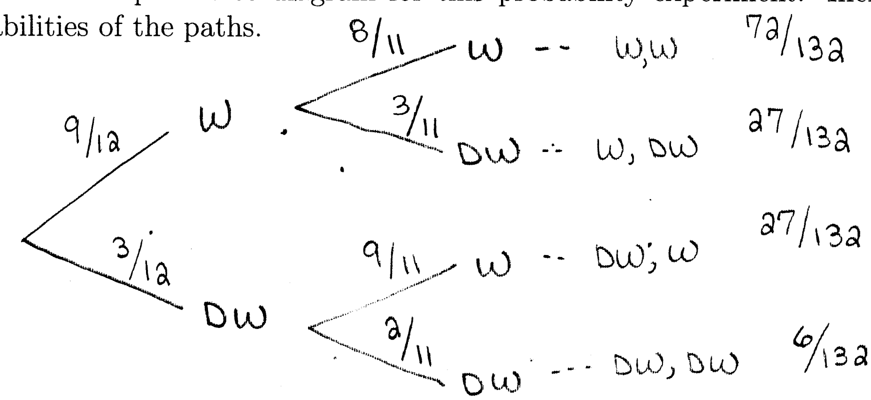
$$\left.\begin{array}{l} \text{(b)} \cdots P(\text{SMOKER}) = \frac{17}{226} \approx 7.5\% \\ \text{(e)} \cdots P(\text{SMOKER} \mid \text{MALE}) = \frac{10}{99} \approx 10.1\% \end{array}\right\}$$

NOT EQUAL

$\Rightarrow$ NOT INDEP.

12. (13 points) In a collection of 12 Apple iPads, there are 3 that do not work. Two iPads are selected at random from the collection (without replacement), and it is determined whether each works or does not work.

(a) Sketch the complete tree diagram for this probability experiment. Include the probabilities of the paths.



(b) In the sample of two, let x represent the number of iPads that do not work. What are the possible values of x?

$X = 0, 1, \text{or } 2$

(c) Determine the probability distribution for x. Write your answer in the form of a table.

	WW	W,DW / DW,W	DW, DW
X	0	1	2
P(x)	$\frac{72}{132}$	$\frac{54}{132}$	$\frac{6}{132}$

(d) Find the mean values of x.

$$\mu = 0\left(\frac{72}{132}\right) + 1\left(\frac{54}{132}\right) + 2\left(\frac{6}{132}\right) = \frac{66}{132} = \boxed{0.5}$$

(e) Find the standard deviation in x.

$$\sigma = \sqrt{0\left(\frac{72}{132}\right) + 1\left(\frac{54}{132}\right) + 4\left(\frac{6}{132}\right) - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{45}{132}} \approx \boxed{0.58}$$

(f) Is it unusual for both iPads in the sample to not work?

Yes, $P(x \geq 2) = \frac{6}{132} \approx 4.5\% < 5\%$

13. (16 points) 71% of Americans believe illegal drugs are a serious problem in the U.S. Suppose a simple random sample of 35 Americans is obtained.

BINOMIAL
$n=35$
$p=0.71$
$q=0.29$

(a) What is the probability that exactly 18 people believe drugs are a serious problem?

$$P(x=18) = \text{binompdf}(35, 0.71, 18)$$
$$\approx 0.0069$$

(b) What is the probability that no fewer than 20 people believe drugs are a serious problem?

$$P(x \geq 20) = 1 - P(x \leq 19)$$
$$= 1 - \text{binomcdf}(35, 0.71, 19) \approx 0.9736$$

(c) A quick check will show you that $P(x = 29) = 0.04691$. Does this indicate the 29 is an unusual value? Explain.

No, you NEED TO USE A CDF TO DETERMINE UNUSUAL VALUES

(d) In the sample of 35, what would be an usually small number of people who think drugs are a serious problem? (Be sure to show your work.)

$$np - 2\sqrt{npq} = 35(0.71) - 2\sqrt{35(0.71)(0.29)} \approx 19.48$$

19 OR FEWER

14. (12 points) The Ford Edge has a mean highway gas mileage of 27 miles per gallon (mpg) with a standard deviation of 3 mpg. A rental car company buys a fleet of 60 of these cars.

CLT
$\mu_{\bar{x}} = 27$
$\sigma_{\bar{x}} = \frac{3}{\sqrt{60}}$

(a) What is the probability that the mean gas mileage of the fleet exceeds 26.5 mpg?

$$P(\bar{x} > 26.5) = \text{normalcdf}\left(26.5, 999999, 27, \frac{3}{\sqrt{60}}\right)$$
$$\approx 0.9016$$

(b) Would it be unusual if the mean gas mileage of the fleet were less than 26 mpg? Show work or explain.

$$P(x < 26) = \text{normalcdf}\left(-999999, 26, 27, \frac{3}{\sqrt{60}}\right) \approx 0.0049 < 5\%$$

YES, UNUSUAL

(c) In order to answer the question in part (a), did you have to assume that the gas mileages were normally distributed? Explain.

No, THE SAMPLE SIZE IS BIG ENOUGH TO ENSURE SAMPLE MEANS ARE NORMALLY DISTRIBUTED.

15. (16 points) The yearly returns of the Dow Jones Industrial Average (DJIA) are approximately normally distributed. In looking at the data since 1975, the DJIA yearly returns have a mean of \$522.68 with a standard deviation of \$1426.02.

(a) What is the probability that a yearly return exceeds \$1500?

$P(x > 1500) = \text{normalcdf}(1500, 999999, 522.68, 1426.02)$

≈ 0.2466

(b) What is the probability that a yearly return is exactly \$525?

$P(x = 525) = \boxed{0}$

(c) In the last 40 years, about how many yearly returns have been between \$1000 and \$2000? (FYI: The actual number is 9.)

$40 * \text{normalcdf}(1000, 2000, 522.68, 1426.02)$

≈ 8.75 About 9

(d) What yearly return is at the 90th percentile?

$\text{invNorm}(0.90, 522.68, 1426.02) \approx 2350.20$

16. (12 points) You would like to determine a 90% confidence interval estimate for the proportion of U.S. voters who approve of President Trump.

(a) What sample size must you use in order to determine a 90% confidence interval estimate with a ±4% margin of error. (According to a Gallup poll, the president's current approval rating is at 46%.)

$$\frac{(z_{\alpha/2})^2 (0.46)(0.54)}{(0.04)^2} = \frac{(1.645)^2(0.46)(0.54)}{(0.04)^2} = 420.1104$$

Use $n = 421$

(b) In a random sample of 128 local voters, you find that 51 approve of the president. Based on your sample, find a 90% confidence interval estimate for the proportion of local voters who approve of the president.

1-PropZInt... $(0.3273, 0.4696)$ or $(32.73\%, 46.96\%)$

(c) What is the margin of error in the interval you found in part (b)?

$$\frac{0.4696 - 0.3273}{2} = 0.07115$$

About 7%

17. (15 points) In a test of ALDI brand batteries, nine batteries were used to power photo flashes. The numbers of flashes required to drain the batteries from 1.5 volts to 0.8 volts were recorded:

638, 645, 636, 651, 639, 649, 654, 627, 644.

You believe that the population mean is greater than 635 flashes, and you intend to test your hypothesis.

(a) State the null and alternative hypotheses.

CLAIM: $\mu > 635$

COUNTER: $\mu \leq 635$ $\Rightarrow$ $H_0: \mu = 635$

$H_1: \mu > 635$

(b) In order to test the claim, will you use a t-test or a z-test? Why?

T-TEST, σ IS NOT KNOWN.

(c) In order to carry out your hypothesis test, what assumption must be made?

THE NUMBERS OF FLASHES REQUIRED TO DRAIN ARE NORMALLY DISTRIBUTED.

(d) Compute the test statistic.

CALCULATOR ... $t \approx 2.685$

(e) Find the P-value and draw a conclusion about your original claim at the level $\alpha = 0.05$. Write your conclusion in a complete sentence.

CALCULATOR... P-VALUE ≈ 0.01386

P-VALUE $< \alpha = 0.05 \Rightarrow$ REJECT H_0.

THE EVIDENCE SUPPORTS THE CLAIM THAT $\mu > 635$.

Here is a larger copy of the ogive associated with problem #1.