

Math 157 - Quiz 8

October 23, 2013

Name key

Score _____

Show all work to receive full credit. Supply explanations when necessary.

1. (10 points) Let $g(x) = x^4 - 4x^3$ on $[-1, 4]$.

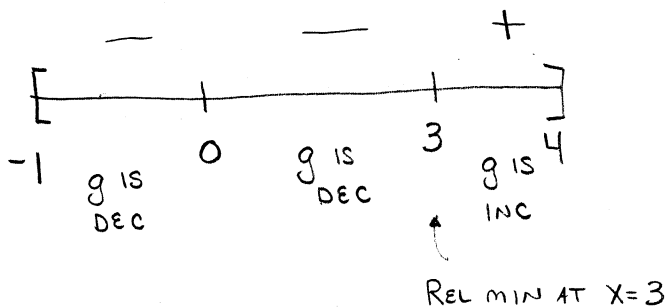
- Find open intervals on which g is increasing/decreasing.
- Identify all relative extrema.
- Identify the absolute extrema.

$$g'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$$

$g'(x)$ DNE NOWHERE

$$g'(x) = 0 \Rightarrow x=0, x=3$$

Signs
of g'



$$g(-1) = 5 \quad \leftarrow \text{ABS MAX}$$

$$g(4) = 0$$

$$g(0) = 0$$

$$g(3) = -27 \quad \leftarrow \text{ABS MIN}$$

g IS INCREASING ON $(3, 4)$

g IS DECREASING ON $(-1, 0) \cup (0, 3)$

$g(3) = -27$ IS A RELATIVE MIN AND THE ABSOLUTE MIN

$g(-1) = 5$ IS THE ABSOLUTE MAX

2. (Bonus: 2 pts ex cred) Let $f(x) = x^{8/3} - 16x^{2/3}$. Find the critical numbers of f .

$$f'(x) = \frac{8}{3}x^{5/3} - \frac{32}{3}x^{-1/3} = \frac{8}{3}x^{-1/3}(x^{6/3} - 4) = \frac{8(x^2 - 4)}{3x^{1/3}}$$

$$f'(x) \text{ DNE} \Rightarrow x = 0$$

$$f'(x) = 0 \Rightarrow x^2 - 4 = 0 \Rightarrow x = \pm 2$$

CRIT. NUMBERS ARE $x = 0, x = 2,$
 $x = -2$

3. (Bonus: 3 pts ex cred) A spherical snowball is placed in the sun. The sun melts the snowball so that its radius decreases $1/4$ in per hour. Find the rate of change of the volume with respect to time at the instant the radius is 4 in.

V = VOLUME AT TIME t

r = RADIUS AT TIME t

$$\frac{dr}{dt} = -\frac{1}{4} \quad \text{FIND } \frac{dV}{dt} \text{ WHEN } r = 4$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

WHEN $r = 4 \dots$

$$\frac{dV}{dt} = 4\pi(4)^2\left(-\frac{1}{4}\right) = -16\pi \frac{\text{in}^3}{\text{hr}}$$