Math 157 - Test 1

September 18, 2013

Name key

Score

Show all work to receive full credit. Supply explanations where necessary.

1. (4 points) Find the x- and y-intercepts of the graph of $y = x^2 - 4x + 3$.

X - INTERCEPTS

$y = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3,\ x = 1$

$\boxed{(3,0),\ (1,0)}$

Y - INTERCEPT

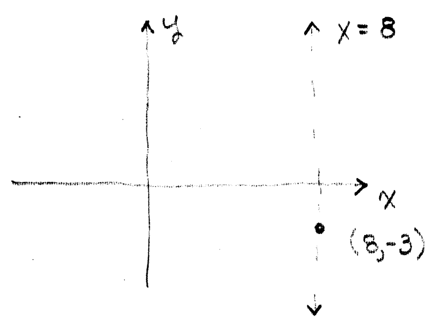
$x = 0$

$y = (0)^2 - 4(0) + 3$

$y = 3$

$\boxed{(0,3)}$

2. (2 points) Write an equation of the vertical line passing through $(8, -3)$.



$\boxed{x = 8}$

3. (5 points) Find $f^{-1}(x)$ if $f(x) = \sqrt{2x - 3}$.

RANGE OF f IS $\{y : y \geq 0\}$

$y = \sqrt{2x-3}$

$y^2 = 2x - 3$

$y^2 + 3 = 2x$

$\dfrac{y^2 + 3}{2} = x$

$$\boxed{f^{-1}(x) = \frac{x^2 + 3}{2},\quad x \geq 0}$$

* DOMAIN OF f^{-1} MUST BE RANGE OF f.

4. (5 points) The supply and demand equations for a handheld video game system are given by

$$p = 240 - 4x$$

$$p = 135 + 3x$$

where p is the price (in dollars) and x is the number of units (in hundreds of thousands).

(a) Which equation is probably the supply equation and which is probably the demand equation? Explain your reasoning.

$p = 135 + 4x$ IS PROBABLY THE SUPPLY EQUATION. → IT IS AN INCREASING FUNCTION OF X.

SINCE $p = 240 - 4x$ IS A DECREASING FUNCTION, IT IS THE DEMAND EQN.

(b) Find the equilibrium point.

$$240 - 4x = 135 + 3x$$

$$105 = 7x$$

$$\frac{105}{7} = x \rightarrow x = 15$$

$$p = 135 + 3(15) = 180$$

$$(x, p) = (15, 180)$$

5. (4 points) Write an equation of the line passing through $(4, 7)$ and perpendicular to $5x + 8y = 13$.

$$8y = -5x + 13$$

$$y = -\frac{5}{8}x + \frac{13}{8}$$

$$m = -\frac{5}{8} \rightarrow m_{\perp} = \frac{8}{5}$$

$$y - 7 = \frac{8}{5}(x - 4)$$

OR

$$y = \frac{8}{5}x + \frac{3}{5}$$

6. (3 points) Let $f(x) = 2x - 1$ and $g(x) = \dfrac{1}{x}$.

(a) Find the composition $g(f(x))$.

$$g(f(x)) = \frac{1}{f(x)} = \frac{1}{2x-1}$$

(b) What is the domain of $g(f(x))$?

$$2x - 1 \neq 0 \Rightarrow x \neq \frac{1}{2}$$

$$\{x : x \neq \tfrac{1}{2}\}$$

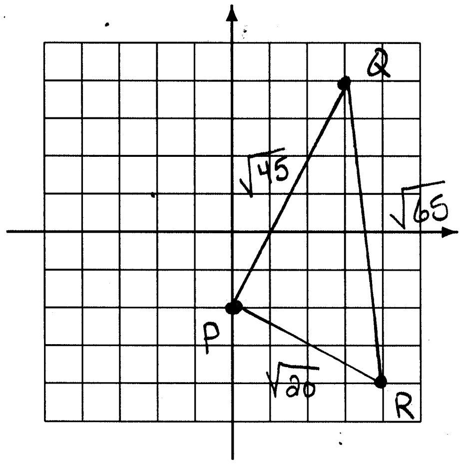
7. (8 points) Plot the points. Then use the distance formula and the Pythagorean theorem to determine whether the points are the vertices of a right triangle.

$$P\,(0,-2),\quad Q\,(3,4),\quad R\,(4,-4)$$

$$\overline{PQ} = \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45}$$

$$\overline{PR} = \sqrt{4^2 + (-2)^2} = \sqrt{16+4} = \sqrt{20}$$

$$\overline{QR} = \sqrt{1^2 + (-8)^2} = \sqrt{1+64} = \sqrt{65}$$



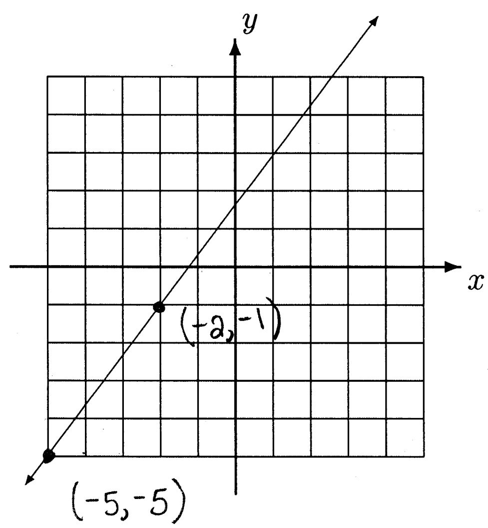
Pythag ...

$$\sqrt{65}^2 = \sqrt{45}^2 + \sqrt{20}^2$$

$$65 = 45 + 20 \quad \text{Yes!}$$

The Δ is a right Δ.

8. (4 points) Find an equation of the line whose graph is shown below.



$$m = \frac{-1-(-5)}{-2-(-5)} = \frac{4}{3}$$

$$\boxed{y+5 = \frac{4}{3}(x+5)}$$

or

$$y = \frac{4}{3}x + \frac{5}{3}$$

9. (5 points) Explain why $\lim_{x \to 2} \frac{|x-2|(x+4)}{x-2}$ does not exist.

X	$\frac{\lvert X-2\rvert (X+4)}{X-2}$
2.1	6.1
2.01	6.01
2.001	6.001

X	$\frac{\lvert X-2\rvert (X+4)}{X-2}$
1.9	-5.9
1.99	-5.99
1.999	-5.999

LIMIT FROM THE LEFT $= -6$ $\neq$ LIMIT FROM RIGHT $= 6$

10. (24 points) Compute each limit. If appropriate, use $+\infty$, $-\infty$, or DNE.

(a) $\lim_{x \to 8^-} (x^2 - 7x + 3)$

$= (8)^2 - 7(8) + 3 = \boxed{11}$

(b) $\lim_{x \to -3} \frac{x^2 + 2x - 3}{x^2 + 4x + 3}$

$0/0$ MORE WORK

$= \lim_{X \to -3} \frac{(X+3)(X-1)}{(X+3)(X+1)} = \lim_{X \to -3} \frac{X-1}{X+1} = \frac{-4}{-2} = \boxed{2}$

(c) $\underbrace{\lim_{x \to 5^+} \frac{x-7}{x-5}}$

$-2/0$ UNBOUNDED

RIGHT OF $X = 5$: $\frac{-}{+} = - \Rightarrow$ $\boxed{\text{LIMIT IS } -\infty}$

(d) $\lim_{x \to 0} \frac{\sqrt{x+9} - 3}{x}$

$\frac{0}{0}$ MORE WORK.

$\cdot \frac{\sqrt{X+9} + 3}{\sqrt{X+9} + 3} = \lim_{X \to 0} \frac{X + \cancel{9} - \cancel{9}}{\cancel{X}(\sqrt{X+9} + 3)} = \lim_{X \to 0} \frac{1}{\sqrt{X+9} + 3}$

$= \frac{1}{\sqrt{9} + 3} = \boxed{\frac{1}{6}}$

11. (5 points) Use a table of values to guess the limit. Your table should include at least six different values of the function near the limit point.

$$\lim_{x \to 1} \frac{x^7 - 1}{x - 1}$$

X	$\frac{X^7-1}{X-1}$
0.9	5.2170
0.99	6.7935
0.999	6.9790
0.9999	6.9979
1.01	7.2135
1.001	7.0210
1.0001	7.0021

IT LOOKS LIKE

$$\lim_{X \to 1} \frac{X^7 - 1}{X - 1} = \underline{\underline{7}}.$$

12. (3 points) Determine where $F(x) = \dfrac{x-3}{\underbrace{x^2 - 9}_{(X-3)(X+3)}}$ is continuous.

$$F(x) = \frac{X-3}{(X-3)(X+3)}$$

F IS NOT DEFINED AT $X = 3$ OR $X = -3$

F IS CONTINUOUS EVERYWHERE EXCEPT $X = 3$ OR $X = -3$

13. (6 points) Find k so that g is continuous at $x = 2$.

$$g(x) = \begin{cases} 8x + k, & x < 2 \\ x^2 + \sqrt{x+2}, & x \geq 2 \end{cases}$$

$$\lim_{X \to 2} g(x) = g(2) \Rightarrow g(2) = (2)^2 + \sqrt{2+2}$$

$$=$$

$$8(2) + k = \lim_{X \to 2^-} g(x)$$

$$\Rightarrow 6 = 16 + k$$

$$\boxed{k = -10}$$

14. (10 points) Let $f(x) = x^2 - 5x + 1$. Use the limit definition of derivative to find $f'(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\left[(x+\Delta x)^2 - 5(x+\Delta x) + 1\right] - \left[x^2 - 5x + 1\right]}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - 5x - 5\Delta x + 1 - x^2 + 5x - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + \Delta x^2 - 5\Delta x}{\Delta x} = \lim_{\Delta x \to 0} (2x + \Delta x - 5)$$

$$= \boxed{2x - 5}$$

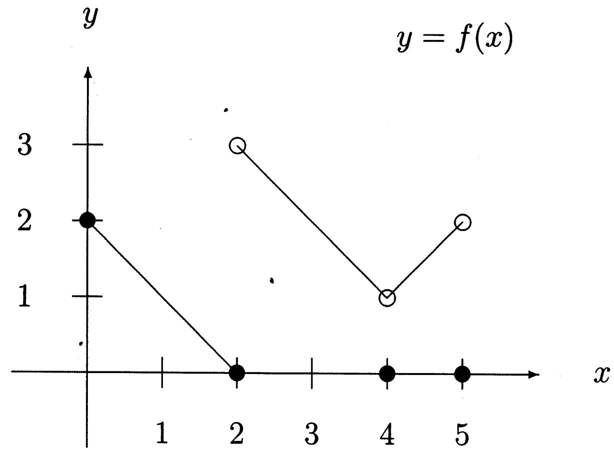
15. (5 points) Let $f(x) = \frac{1}{x+2}$. It can be shown that $f'(x) = \frac{-1}{(x+2)^2}$. Use this information to find an equation of the line tangent to the graph of f at the point where $x = 3$.

$$m = f'(3) = \frac{-1}{5^2} = \frac{-1}{25}$$

$$x = 3 \Rightarrow y = f(3) = \frac{1}{5}$$

$$\boxed{y - \frac{1}{5} = -\frac{1}{25}(x-3)}$$

16. (7 points) Referring to the graph shown below, determine each of the following or explain why it does not exist.



(a) $\lim_{x \to 4} f(x)$ = $\boxed{1}$

(b) $f'(1)$ = SLOPE AT X = 1 = $\boxed{-1}$

(c) $\lim_{x \to 2^-} f(x)$ = $\boxed{0}$

(d) $f(5)$ = $\boxed{0}$

(e) $\lim_{x \to 2^+} f(x)$ = $\boxed{3}$

(f) The x-coordinate of a point at which $f'(x) > 0$ $\boxed{X = 4.5}$

SLOPE IS POSITIVE

(g) The x-coordinate of a point at which f is NOT continuous.

$\boxed{X = 2}$